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Part I.

By P. COMRIE
M.A., B.Sc., F.R.S.E.
and
W. WOODBURN

Without Answers,

CHAMBERS'S NEW COMMERCIAL ARITHMETIC

BY

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PART I

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1916

PREFACE.

THE object of the work of which this volume forms Part I. is to provide a course of Commercial Arithmetic suitable for Evening Continuation Classes, students on the commercial side of secondary schools, and private students. The book presupposes a knowledge of ordinary arithmetic up to and including vulgar and decimal fractions. Its main feature is the constant use of short methods. The commercial student must not only get the correct answer, but must get it in the shortest possible time. With this end in view, mental working is insisted on from the beginning, and contracted multiplication and division are introduced early and used throughout the course. Full and lucid explanations of the rules and of the principles underlying them are given, and numerous examples of a practical nature are provided.

The examination papers at the end of the book have been selected from the Leaving Certificate Examination Papers with the special permission of the Controller of H.M. Stationery Office. To him and to several gentlemen who have read through the proofs and offered valuable suggestions the publishers and the author offer their grateful thanks.

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CHAMBERS'S NEW COMMERCIAL ARITHMETIC.

PART I.

TABLES OF BRITISH WEIGHTS AND MEASURES.

AVOIRDUPOIS WEIGHT.

16 drams (dr.)	= 1 ounce (oz.).
16 oz.	= 1 pound (lb.).
28 lb.	= 1 quarter (qr.).
4 qr.	= 1 hundredweight (cwt.).
20 cwt.	= 1 ton.

14 lb.	= 1 stone (st.).
112 lb.	= 1 cwt.
2240 lb.	= 1 ton.

LINEAR MEASURE.

12 inches (in.)	= 1 foot (ft.).
3 ft.	= 1 yard (yd.).
5½ yd.	= 1 rod or pole (pc. or pl.).
40 po.	= 1 furlong (fur.).
8 fur.	= 1 mile (ml.).

100 links (lk.)	= 1 chain (ch.).
22 yd.	= 1 chain.
10 ch.	= 1 furlong.
220 yd.	= 1 furlong.
1760 yd.	= 1 mile.
5280 ft.	= 1 mile.

SQUARE MEASURE.

144 sq. inches	= 1 sq. ft.
9 sq. ft.	= 1 sq. yd.
30¼ sq. yd.	= 1 sq. po. or perch.
40 sq. po.	= 1 rood (ro.).
4 ro.	= 1 acre (ac.).
640 ac.	= 1 sq. mile.
484 sq. yd.	= 1 sq. chain.
10 sq. ch.	= 1 acre.
4840 sq. yd.	= 1 acre.

CUBIC MEASURE.

1728 cub. in.	= 1 cub. ft.
27 cub. ft.	= 1 cub. yd.

Note.—1 cubic foot of pure water weighs 1000 oz.

LIQUID MEASURE.

4 gills	= 1 pint (pt.).
2 pt.	= 1 quart (qt.).
4 qt.	= 1 gallon (gal.).
36 gal.	= 1 barrel.

Note.—In the north of England and in parts of Scotland a half-pint is called a gill.

CORN OR DRY MEASURE.

4 gills	= 1 pint (pt.).
2 pt.	= 1 quart (qt.).
4 qt.	= 1 gallon (gal.).
2 gal.	= 1 peck (pk.).
4 pk.	= 1 bushel (bus.).
8 bus.	= 1 quarter (qr.).

TROY WEIGHT.

24 grains (gr.)	= 1 pennyweight (dwt.).
20 dwt.	= 1 ounce (oz.).
12 oz.	= 1 pound (lb. tr.).

1 lb. (tr.).	contains 5760 grains.
1 lb. (avoird.)	" 7000 grains.

LONG TOTS AND CROSS TOTS (SIMPLE).

The adding of long columns and rows of figures is an every-day requirement in business. As a preliminary step

the combinations making 10, and the sum of every pair of figures from 1 to 9, should be known thoroughly. The student should be able to take in at a glance pairs of figures (or small groups of figures) making 10, 11, 12, &c., and add these on at one step. Use as few words as possible in the process. The right-hand column in 1 (*a*) below might be added thus—5, 11, 21, 36, 44. Results should be checked by adding from the top downwards.

Exercise 1.

Part I.—In each of the following six sets, add together (i.) the vertical columns, (ii.) the horizontal rows, and (iii.) each set of answers. The two results obtained in (iii.) will, of course, be the same if the working is correct.

Part II.—Treat sets 1 and 2 as one exercise (of sixteen lines), and proceed as in Part I. Similarly, work sets 3 and 4 together as an exercise of twenty lines, and sets 4 and 5 together as an exercise of twenty-four lines.

Part III.—Treat sets 3, 4, and 5 as one exercise of thirty-four lines.

Set. 1.

	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
(1)	991258	873529	126455	526728	484846
(2)	409418	278750	478736	669338	971872
(3)	946927	537976	563026	959265	521897
(4)	185494	851197	715298	735432	920442
(5)	609816	776599	323002	562251	821796
(6)	227716	237583	804447	627388	479805
(7)	475332	988419	771868	362231	681467
(8)	727113	324462	323911	302582	719473

Set 2.

(1)	423561	375186	584176	732945	781963
(2)	725914	247943	828714	946813	58347
(3)	837063	894261	254671	387246	289548
(4)	478665	582734	895186	748139	4379
(5)	397164	697843	628573	459876	684
(6)	504283	485718	321458	689528	71593
(7)	982115	149248	540287	829637	832765
(8)	674819	963123	764158	534946	48

SET 3.

	(a)	(b)	(c)	(d)	(e)
(1)	825134	7	846518	717103	676371
(2)	4956	89	75429	775940	731682
(3)	87	453	8635	823405	781593
(4)	91482	7825	478	824393	794205
(5)	6	29147	69	904687	802186
(6)	784	385426	8	914273	815431
(7)	5938	94728	527	932166	846257
(8)	429876	5819	604718	954273	870453
(9)	72534	43128	76259	969988	918298
(10)	8713	5984	6451	985014	946183

SET 4.

(1)	184763	208968	32320	57557	1031819
(2)	193485	225783	33764	58209	1054345
(3)	204763	238152	34742	58808	1151997
(4)	215840	247995	36244	61092	1255462
(5)	230122	262168	41142	62044	1469601
(6)	251638	295805	43875	63144	1480935
(7)	285746	304160	45775	66705	1679492
(8)	293019	315293	48590	68340	1810328
(9)	305148	321864	49809	69206	1916696
(10)	329706	352189	50723	69800	2073392

SET 5.

(1)	8207	9073	10144	11286	14932
(2)	7212	7812	8059	9315	9863
(3)	3579	5334	6495	7236	8054
(4)	229390	236240	251048	263415	287931
(5)	178352	179205	182536	198554	214036
(6)	273594	282142	290422	305168	327841
(7)	415465	421964	436289	471802	504179
(8)	22030	31212	35406	38537	40236
(9)	38665	39030	42187	46049	48152
(10)	4178	7590	8301	9568	9723
(11)	52463	67419	83246	91428	98614
(12)	589	612	843	916	990
(13)	56	71	83	92	97
(14)	149	236	541	802	910

Set 6.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	Totals.
(1)	37469	8432	25438	43861	72964	53869	9846	37258	69043	875	
(2)	8135	76157	7684	984	8369	6478	86059	93684	79	84368	
(3)	19056	58614	932	37659	63984	19380	674	3769	81549	8739	
(4)	8364	19840	46378	85438	7268	86493	92847	76589	3987	63915	
(5)	749	675	8615	674	95736	6159	6298	9836	8769	78547	
(6)	84376	35486	98157	93859	8649	83684	19358	17698	678	76938	
(7)	5918	9643	6279	67436	738	95973	96487	98432	53849	63947	
(8)	65489	51735	56398	92768	17648	9846	37459	278	65748	36493	
(9)	38617	62847	49216	9689	8398	73916	8267	6472	8239	9875	
(10)	5849	58639	17385	85643	94673	8759	98	83619	36158	47394	
(11)	64538	2746	4639	7685	86497	37648	74683	74835	8649	9748	
(12)	9671	38485	85478	63594	2938	729	38974	9647	46387	15469	
(13)	58169	9261	6315	867	47489	9384	8615	83159	38469	3856	
(14)	98470	53637	90237	74639	9638	36859	42617	92648	9437	62549	
(15)	86175	8741	58697	86	82379	9284	3849	3476	836	3685	
(16)	29460	295	29438	46074	19084	67395	98639	63984	91547	92467	
(17)	3859	46	9846	63958	8379	8465	7483	5367	8469	7386	
(18)	92617	93768	37618	4639	62590	72915	82678	29846	58398	62749	
(19)	48739	23584	5893	82176	18769	6738	9375	3738	827	6385	
(20)	9814	670	76539	65397	9237	93649	82647	65498	7489	83564	
Totals.											Grand Total.

LONG TOTS AND CROSS TOTS (COMPOUND).

The following examples, if worked through in the order suggested, will furnish a large number of well-graduated exercises.

Exercise 2.

Set 1.

Part I.—In each of the following examples, A, B, C, D, E, F, G, H, I, add together (i.) the vertical columns, (ii.) the horizontal rows, and (iii.) each set of answers. The two results obtained in (iii.) will, of course, be the same if the working is correct.

Part II.—Treat the examples A and B as one exercise (of ten lines), and proceed as in Part I. Similarly, work examples B and C, D and E, and E and F together as exercises of ten lines, and G and H, and H and I together as exercises of twelve lines.

Part III.—Treat the examples A, B, and C as one exercise (of fifteen lines), and proceed as in Parts I. and II. In the same way, work examples D, E, and F together as an exercise of fifteen lines, and G, H, and I together as an exercise of eighteen lines.

		(*a*)			(*b*)			(*c*)			(*d*)		
		£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
A.	(1)	12	12	8	7	13	0	2	1	6	1	12	10
	(2)	3	0	0	1	8	5	6	4	4	8	7	2
	(3)	11	16	0	30	16	1	15	17	3	25	2	2
	(4)	22	11	5	6	9	2	75	11	9	11	18	8
	(5)	2	19	4	18	5	4	102	8	6	17	10	6
B.	(6)	9	5	6	3	3	9	7	10	10	8	2	7
	(7)	10	9	4	3	15	5	3	5	7	3	12	8
	(8)	6	7	6	2	10	9	5	9	3	2	2	7
	(9)	14	12	2	1	10	11	22	14	8	48	12	6
	(10)	6	10	8	1	16	3	0	11	8	1	2	10
C.	(11)	15	7	7	8	2	2	4	19	5	54	19	4
	(12)	14	17	4	18	12	1	3	13	9	60	1	1
	(13)	6	7	3	4	4	4	11	12	10	1	7	3
	(14)	5	4	6	18	0	5	77	10	6	78	13	7
	(15)	4	4	3	3	17	10	0	18	2	16	18	10

		(a) £ s. d.	(b) £ s. d.	(c) £ s. d.	(d) £ s. d.
D.	(1)	22 15 4	17 16 3	25 11 10	58 13 6
	(2)	13 7 10	21 17 8	62 18 4	72 19 8
	(3)	9 4 6	62 16 4	10 0 3	85 11 7
	(4)	5 11 10	51 14 9	41 15 7	39 16 11
	(5)	84 16 7	62 19 8	19 18 11	46 13 9
E.	(6)	212 9 3	513 7 11	127 16 6	430 11 5
	(7)	371 8 4	623 9 7	341 13 7	269 14 7
	(8)	613 11 10	204 0 10	624 11 8	394 12 10
	(9)	127 13 9	307 10 3	270 14 7	563 17 8
	(10)	865 17 6	937 16 8	935 16 10	925 14 8
F.	(11)	729 16 4	614 9 11	453 8 11	756 18 4
	(12)	816 11 3	623 7 9	637 9 9	293 17 6
	(13)	162 17 11	184 11 7	411 11 11	847 16 8
	(14)	203 0 7	264 9 0	807 10 10	671 12 10
	(15)	368 5 9	935 16 8	725 18 11	938 16 7

		(a) £ s. d.	(b) £ s. d.	(c) £ s. d.	(d) £ s. d.
G.	(1)	156 18 $4\frac{1}{2}$	734 18 $11\frac{1}{4}$	583 17 $6\frac{1}{2}$	810 12 $8\frac{1}{4}$
	(2)	73 12 $9\frac{3}{4}$	231 7 $10\frac{1}{2}$	79 18 $9\frac{1}{2}$	527 13 $7\frac{1}{2}$
	(3)	48 17 $6\frac{1}{4}$	88 9 $6\frac{3}{4}$	93 12 $6\frac{1}{2}$	895 17 $9\frac{3}{4}$
	(4)	235 11 $10\frac{1}{2}$	105 16 $9\frac{1}{2}$	314 15 $2\frac{3}{4}$	64 16 6
	(5)	96 12 $1\frac{1}{2}$	603 15 $8\frac{3}{4}$	52 14 $3\frac{1}{2}$	83 12 9
	(6)	8 16 $11\frac{1}{2}$	59 12 $3\frac{1}{4}$	67 16 $8\frac{1}{2}$	153 18 $4\frac{3}{4}$
H.	(7)	126 18 $3\frac{1}{2}$	295 16 $8\frac{1}{2}$	619 13 $6\frac{3}{4}$	453 12 $7\frac{3}{4}$
	(8)	340 11 $4\frac{1}{2}$	69 7 $9\frac{3}{4}$	857 8 $8\frac{1}{2}$	18 19 $2\frac{1}{2}$
	(9)	31 3 9	843 18 $6\frac{1}{4}$	396 17 $11\frac{1}{4}$	264 5 11
	(10)	516 13 $7\frac{3}{4}$	729 19 11	84 12 $10\frac{3}{4}$	172 8 $10\frac{1}{4}$
	(11)	64 15 $6\frac{1}{4}$	864 10 10	729 9 $8\frac{1}{2}$	23 14 $6\frac{3}{4}$
	(12)	236 18 $1\frac{1}{2}$	98 13 $8\frac{1}{2}$	986 15 $7\frac{1}{4}$	89 17 $8\frac{1}{4}$
I.	(13)	267 15 $3\frac{1}{2}$	567 12 $9\frac{1}{2}$	729 16 $9\frac{1}{2}$	12 2 $4\frac{3}{4}$
	(14)	86 13 $6\frac{3}{4}$	834 17 6	846 3 $11\frac{1}{4}$	375 6 $7\frac{1}{3}$
	(15)	928 14 $7\frac{1}{3}$	86 14 $9\frac{1}{4}$	78 17 6	8 19 $3\frac{1}{4}$
	(16)	86 17 $9\frac{1}{4}$	367 8 $6\frac{1}{4}$	539 13 $8\frac{1}{4}$	236 14 10
	(17)	367 15 6	935 15 $9\frac{1}{2}$	67 18 $6\frac{1}{2}$	592 12 $6\frac{3}{4}$
	(18)	87 10 11	846 7 $3\frac{1}{4}$	749 16 $11\frac{1}{4}$	85 17 $3\frac{1}{4}$

Set 2.

Part I.—Treat examples K and L as separate exercises.

Part II.—Treat both examples together as one exercise of thirty lines.

		£	s.	d.	£	s.	d.	£	s.	d.
K.	(1)	73994	4	5½	66806	12	6½	140800	16	11½
	(2)	55521	12	10½	20633	4	2½	76154	17	0
	(3)	53302	7	3¾	...			53302	7	3¾
	(4)	60475	9	11½	19673	2	0	80148	11	11¾
	(5)	3912	10	0	...			3912	10	0
	(6)	15525	0	0	...			15525	0	0
	(7)	87070	3	1¾	6363	19	10¾	93434	2	11½
	(8)	31364	13	9½	15293	1	11½	46657	15	8½
	(9)	40693	18	9	7331	18	3½	48025	17	0½
	(10)	247997	3	5½	124197	10	11¾	372194	14	4¾
	(11)	5244	1	11¼	...			5244	1	11¼
	(12)	214263	18	5¾	188767	6	10½	403031	5	3½
	(13)	120261	8	0	16938	5	3¼	137199	13	3¼
	(14)	34925	7	2½	30997	16	2½	65923	3	4½
	(15)	66545	19	11¼	8877	0	0	75422	19	11¾
L.	(1)	73821	18	6½	52853	16	7½	26538	17	9
	(2)	9875	4	9¾	9619	13	8	87395	18	10½
	(3)	156743	8	7	36159	18	6	67	17	9¾
	(4)	96397	13	10½	74632	15	8½	36239	8	7½
	(5)	84617	19	8	9385	16	10½	91568	16	11¼
	(6)	76432	18	9½	15647	3	6	7269	14	8
	(7)	936174	17	5¼	9543	15	8½	91564	16	7¾
	(8)	86349	18	7	368159	14	7¾	67398	14	9½
	(9)	7458	13	10½	9156	17	9	481367	7	6½
	(10)	91647	12	9¾	86432	16	10¼	9285	16	3¼
	(11)	86439	11	10	9615	19	3½	63715	14	11½
	(12)	57368	14	7¼	61537	8	6½	8617	19	10
	(13)	186359	8	6½	9158	16	10¾	63845	17	6¾
	(14)	768	15	9	63987	12	9½	81593	16	10½
	(15)	4639	12	11½	8649	17	6¾	7369	7	9½

SET 3.

The following is the expenditure on account of an estate from 1856 to 1869 inclusive:

	Rates and Taxes.	Repairs.	Improvements.	Drainage.	Miscellaneous.	Land-steward.	Total for Year.
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	
1856	67 19 2½	49 7 10	831 0 0	185 15 3½	23 3 1	15 10 0	
1857	41 2 7½	196 13 7	654 2 9	102 17 8	27 11 7½	70 0 7	
1858	51 8 6	127 15 4	54 19 7	244 16 2	30 17 4	18 11 0	
1859	73 11 6	96 4 0	350 8 7½	233 6 2½	22 6 0	14 4 1	
1860	56 3 6	143 19 9	206 15 6½	274 1 10½	20 5 2	49 15 3	
1861	48 11 0	220 12 2½	42 9 0	66 4 10	25 16 8	15 12 0	
1862	50 11 0	157 1 2	273 0 1½	272 2 2	24 11 5	25 1 6	
1863	65 6 8	116 8 10½	...	80 17 1	34 12 9	22 15 0	
1864	60 1 10½	169 14 0	...	...	33 14 5	23 0 0	
1865	70 18 2½	86 17 9	86 15 7	...	40 16 7	46 17 3	
1866	59 14 10	119 8 11½	48 19 6½	...	29 17 6½	18 19 7	
1867	67 7 11½	97 14 10	...	78 14 11	95 18 4	26 4 9	
1868	73 18 6	86 13 7	378 17 9	29 18 10	27 8 9½	35 6 7	
1869	66 17 9	178 9 9	58 16 7	98 15 6	34 16 7½	30 18 8	
Total under each Head.							Grand Total.

Find, without rearranging the figures, the total expenditure for each year, the total expenditure under each head for the fourteen years, and the entire expenditure on account of the estate during that time.

SET 4. Extend (in the manner shown in the example) the following record of transactions extracted from a merchant's Day-book, and find the grand total of the sales recorded.

Example given :

JONES & SON, Inchkeith.	£	s.	d.	£	s.	d.
12 doz. Shakespeare's Plays @ 12s. per doz.	7	4	0			
60 " Poetry for Recitation @ 1s. "	3	0	0			
8 vols. Eng. Dictionary @ 12s. 6d. per vol.	5	0	0			
16 doz. Commercial Arith. @ 8s. per doz.	6	8	0	21	12	0

JULY 1, 1899.

	£	s.	d.	£	s.	d.
SMITH & Co., Glasgow.						
12 doz. Classics @ 2s. per doz.						
15 " " @ 4s. "						
6 " " @ 9s. "						
3 " " @ 12s. "						
BROWN & Co., Aberdeen.						
8 doz. Reward Books @ 15s. per doz.						
4 " " @ 6s. "						
6 " " @ 12s. "						
2 " " @ 18s. "						
5 " Elocutions @ 50s. "						
BRANDON & SONS, Manchester.						
12 doz. Plays @ 12s. per doz.						
9 " " @ 15s. "						
5 " " @ 18s. "						
3 sets Encyclopædia, 10 vols. ½ morocco @ 150s. per set.						
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10 doz. Easy Stories for Infants @ 6s. per doz.						
6 " Fluent Primers @ 4s. "						
4 " New Reciters @ 30s. "						
HUNTER, J. G., Leeds.						
14 doz. Journal Parts @ 8s. per doz.						
1 " Gazetteer Volumes @ 6s. per vol.						
1 " Biographical Dict. @ 10s. 6d. "						
1 " English Dict. @ 12s. 6d. "						
Grand Total = £						

SET 5. Check the following Balance-Sheet by adding up both columns and making them agree :

1899.								
Jan. 1	Bank.....*Dr.*	£796	8	10				
	Cash..... "	33	2	9				
	J. Adams..... "	47	5	4				
	Bills Receivable..... "	151	4	11				
	Wool..... "	1250	0	0				
	To Capital.....				£2278	1	10	
"	Capital.....*Dr.*	1045	6	3				
	To T. Ellis (Loan at 5 per cent.).....				630	0	0	
	" Bills Payable.....				207	3	9	
	" F. Greenwood.....				158	2	6	
	" Rent outstanding (H. Knight).....				50	0	0	
"	T. Ellis.....*Dr.*	30	0	0				
"	Rent outstanding (H. Knight)..... "	50	0	0				
5	J. Halse..... "	151	4	11				
10	Bills Payable..... "	207	3	9				
11	F. Greenwood..... "	142	6	0				
14	Capital..... "	100	0	0				
"	To Bank.....				680	14	8	
31	Bank.....*Dr.*	589	19	7				
2	To B. James.....				125	6	0	
3	" Bills Receivable.....				151	4	11	
21	" J. Adams.....				200	0	0	
23	" J. Halse.....				113	8	8	
2	B. James.....*Dr.*	128	10	7				
12	W. Heath (for short weight).	12	0	0				
15	J. Adams.....*Dr.*	436	9	5				
31	To Wool.....				577	0	0	
"	Wool.....*Dr.*	321	7	6				
4	To W. Heath.....				321	7	6	
2	Discount.....*Dr.*	3	4	7				
"	To B. James.....				3	4	7	
11	F. Greenwood.....*Dr.*	15	16	6				
"	To Discount.....				15	16	6	
"	Cash.....*Dr.*	47	5	4				
4	To J. Adams.....				47	5	4	
12	W. Heath.....*Dr.*	309	7	6				
	To Bills Payable.....				309	7	6	
	Totals =							

SHORT METHODS IN MULTIPLICATION.

NOTE.—*The examples in this section should be worked mentally.*

To multiply by 10, 100, 1000, &c. Add as many 0's to the multiplicand as there are 0's in the multiplier. If the multiplicand is a decimal, move the decimal point one, two, three, &c. places to the right.

For example, $8573 \times 1000 = 8573000$.

$23{\cdot}765 \times 100 = 2376{\cdot}5$.

To multiply by any multiple of 10, 100, 1000, &c. Multiply by 10, 100, or 1000, and then by the given multiple.

For example, $935{\cdot}6 \times 800 = 93560 \times 8 = 748480$.

1. Multiply 743, 83·27, 5·976, ·0213 by 10, 100, 1000, 70, 500, 7000.

To multiply by 9. Place a 0 to the right of the multiplicand, and from the number thus formed subtract the multiplicand.

For example, $857 \times 9 = 8570 - 857 = 7713$.

When we place a 0 to the right of the multiplicand we multiply it by 10, or take it 10 times. But this is once too often, and, therefore, if we subtract the original number we have left 9 times the multiplicand.

To multiply by 99, 999, 9999, &c. Add as many 0's to the multiplicand as there are 9's in the multiplier, and subtract the multiplicand.

For example, $763 \times 999 = 763000 - 763 = 762237$.

This evidently follows from what has been already said.

In a similar manner, we can multiply by such numbers as 98, 97, 96, &c., and 998, 997, 996, &c., by adding the necessary number of 0's, and subtracting 2, 3, 4, or more times the multiplicand, as the case may be.

For example, $371 \times 96 = 37100 - (4 \times 371) = 37100 - 1484 = 35616$.

2. Multiply 1823, 21·97, 9·564, ·0278 by 9, 99, 999, 98, and 997.

To multiply by 11. Place a 0 to the right of the multiplicand, and to the result add the multiplicand itself.

For example, $6318 \times 11 = 63180 + 6318 = 69498$.

By placing a 0 to the right of the multiplicand we multiply it by 10, or take it 10 times. Therefore, by adding the multiplicand to this result we obtain 11 times the multiplicand.

The readiest way of multiplying by 11 is to write down the first figure on the right of the multiplicand (the 8) as the first figure in the product. For the second figure of the product add 8 and 1, for the third 1 and 3, and so on, carrying where necessary. For the last figure bring down the figure on the left (the 6), having carried to it if necessary.

To multiply by 101, 1001, &c. Multiply by 100, 1000, &c. (that is, add two, three, or more 0's, as the case may be, to the multiplicand), and to the result add the multiplicand itself.

For example, $365 \times 1001 = 365000 + 365 = 365365$.

This evidently follows from what has gone before.

In a similar manner, we can multiply by such numbers as 102, 103, 104, &c., and 1002, 1003, 1004, &c., by affixing the necessary number of 0's, and then adding 2, 3, 4, or more times the multiplicand, as the case may be.

For example, $258 \times 104 = 25800 + 1032 = 26832$.

3. Multiply 9387, 97·21, 4·532, ·0376 by 11, 101, 1001.

To multiply by 5. Place a 0 to the right of the multiplicand, and divide by 2.

For example, $36173 \times 5 = 361730 \div 2 = 180865$.

By adding a 0 we multiply by 10 instead of by 5—that is, our answer is twice as large as it should be. We therefore divide by 2.

To multiply by 25. Add two 0's to the right of the multiplicand, and divide by 4.

For example, $365 \times 25 = 36500 \div 4 = 9125$.

By adding two 0's we multiply by 100 instead of by 25—that is, our answer is 4 times too large. We therefore divide by 4.

To multiply by any fraction of 100. Add two 0's to the right of the multiplicand, and multiply by the fraction.

For example, $385 \times 75 = 38500 \times \frac{3}{4} = \frac{115500}{4} = 28875$.

4. Multiply 7439, 1·893, 74·6, ·0853 by 5, 25, and 75.

To multiply by 125. Add three 0's to the right of the multiplicand, and divide by 8.

For example, $7315 \times 125 = 7315000 \div 8 = 914375$.

By adding three 0's we multiply by 1000—that is, our answer is 8 times too large. We therefore divide by 8.

Similarly, we can *multiply by* 625 by adding four 0's and dividing by 16, because $10000 \div 16 = 625$.

To multiply by 250. Add three 0's to the right of the multiplicand, and divide by 4.

For example, $289 \times 250 = 289000 \div 4 = 72250$.

By adding three 0's we multiply by 1000—that is, our answer is 4 times too large. We therefore divide by 4.

Another method: Place a 0 to the right of the multiplicand, and multiply by 25. The reason is obvious.

To multiply by any fraction of 1000. Add three 0's to the right of the multiplicand, and multiply by the fraction.

For example, $748 \times 625 = 748000 \times \frac{5}{8} = \frac{3740000}{8} = 467500$.

5. Multiply 7652, 42·96, ·8472, 5·432 by 125, 625, 375, and 875.

In multiplying by numbers ending in 5 it is sometimes convenient to double the multiplier and halve *either* the multiplicand *or* the product.

For example, (1) $364 \times 35 = 182 \times 70 = 12740$;

(2) $289 \times 35 = (289 \times 70) \div 2 = 20230 \div 2 = 10115$.

6. Multiply 3467, 81·29, ·0384, 341·6 by 35, 45, and 55.

To multiply together any two numbers between 10 and 20. Add the whole of one number to the units digit of the other, multiply the sum by 10, and add the product of the two units digits.

For example, $13 \times 19 = (13 + 9) \times 10 + (3 \times 9) = 220 + 27 = 247$.

7. Multiply 14, 17, 13, 19, 16, by 13, 15, 18, 17, 19.

To multiply by any number from 11 to 19 (inclusive). Multiply in turn each figure of the multiplicand (beginning with the units figure) by the units digit in the multiplier, and add to each result (beside the carrying figure) the digit to the right of the one you are multiplying in the multiplicand.

For example, 3617×18.

ORDINARY METHOD.	NEW METHOD.
3617	$3617 \times 18 = 65106$
18	The partial products being obtained mentally thus :
28936	$8 \times 7 \quad +0 = 5 \mid 6$
3617	$8 \times 1 + 5 + 7 = 2 \mid 0$
65106	$8 \times 6 + 2 + 1 = 5 \mid 1$
	$8 \times 3 + 5 + 6 = 3 \mid 5$
	$8 \times 0 + 3 + 3 = \quad \mid 6$

This method may be used to multiply by numbers 101 to 109 (or 1001 to 1009), &c., where we add the digit two (or three) places to the right of the one we are multiplying in the multiplicand.

For example, $87542 \times 107 = 9366994$.

The partial products are obtained mentally thus :

$$\begin{array}{ll} 7 \times 2 \quad +0 = 1 & 4 \\ 7 \times 4 + 1 + 0 = 2 & 9 \\ 7 \times 5 + 2 + 2 = 3 & 9 \\ 7 \times 7 + 3 + 4 = 5 & 6 \\ 7 \times 8 + 5 + 5 = 6 & 6 \\ 7 \times 0 + 6 + 7 = 1 & 3 \\ 7 \times 0 + 1 + 8 = & 9 \end{array}$$

8. Multiply 4952, 743·6, ·0281, 4·683 by 13, 16, 19, 102, 108, 1007, and 1009.

Multiplication by factors. Where the multiplier is composed of factors it is often convenient to multiply by each factor in turn.

For example, $879 \times 63 = 879 \times 7 \times 9 = 6153 \times 9 = 55377$.

This method can often be usefully employed in written multiplication. For example, to multiply 93815 by 1089.

93815	
1089	
844335	(1) = 9 times multiplicand.
10132020	(2) = 120 times (1) = 1080 times multiplicand.
102164535	= 1089 times multiplicand.

9. Multiply 849735 by 287, 369, 72189, and 749 in shortest way.

EXERCISE 3.

Multiply in the shortest way:

(1) 28674 by 11, 101, 125
(2) 31568 by 25, 99, 35
(3) 82751 by 17, 997, 250
(4) 36715 by 125, 1005, 13
(5) 75823 by 104, 25, 98
(6) 62174 by 250, 19, 1003
(7) 31768 by 25, 16, 97
(8) 47369 by 125, 15,102
(9) 63971 by 250, 18, 45
(10) 36759 by 125, 35, 998

Multiply (using factors):

(11) 83569 by 24, 56, 72
(12) 31763 by 55, 84, 132
(13) 31763 by 729, 637, 364
(14) 38465 by 819, 968, 777

SHORT METHODS IN DIVISION.

NOTE.—*The examples in this section should be worked mentally.*

To divide by 10, 100, 1000, &c. Cut off as many figures at the right of the dividend as there are 0's in the divisor.

For example, $817563 \div 100 = 8175{\cdot}63$.

1. Divide 84376, 87537·6, 128·34, 9·876 by 10, 100, and 1000.

To divide by 5. Multiply the dividend by 2, and cut off *one* figure at the right of the result (that is, divide it by 10).

For example, $6371 \div 5 = 12742 \div 10 = 1274{\cdot}2$.

In this method we multiply both dividend and divisor by 2, so as to make the latter the convenient number 10.

2. Divide 18375, 74289, 3476·3, 89·375, 6·1284 by 5.

To divide by 25. Multiply the dividend by 4, and cut off *two* figures at the right of the result (that is, divide it by 100).

For example, $36715 \div 25 = 146860 \div 100 = 1468{\cdot}6$.

We multiply both dividend and divisor by 4, so as to make the latter the convenient number 100.

3. Divide 743875, 29856, 4378·5, 187·425, 9·4638 by 25.

To divide by 125. Multiply the dividend by 8, and cut off *three* figures at the right of the result (that is, divide it by 1000.)

For example, $629318 \div 125 = 5034544 \div 1000 = 5034{\cdot}544$.

We multiply both dividend and divisor by 8, so as to make the latter the convenient number 1000.

4. Divide 784625, 189753, 2978·42, 436·974, 8·4735 by 125.

To divide by 250. Multiply the dividend by 4, and cut off *three* figures at the right of the result (that is, divide it by 1000).

For example, $371258 \div 250 = 1485032 \div 1000 = 1485{\cdot}032$.

We multiply both dividend and divisor by 4, so as to make the latter the convenient number 1000.

5. Divide 54375, 82976, 7438·5, 184·265, 4·238 by 250.

In dividing by numbers ending in 5 it is sometimes convenient to double both dividend and divisor, and *halve the remainder* if there be any.

For example, $361758 \div 35 = 723516 \div 70 = 10335\frac{16}{70}$, *or* $10335 + 33$.

6. Divide 68472, 3456·6, 29·376 by 15, 35, and 45.

Exercise 4.

Divide in the shortest way:

(1) 368179 by 10, 25, 35
(2) 584321 by 125, 100, 15
(3) 862370 by 45, 1000, 125
(4) 732189 by 25, 1000, 55
(5) 729165 by 100, 125, 15
(6) 358604 by 250, 10, 35
(7) 361573 by 125, 100, 250
(8) 921500 by 25, 45, 250
(9) 921586 by 35, 250, 5
(10) 821697 by 70, 125, 55

CONTRACTED METHODS FOR LONG DIVISION.

(1) **By factors.** Where the divisor is composed of factors it is often convenient to divide by each factor in turn. For example, to divide 361743 by 56, divide by 8 and 7, the factors of 56.

8)361743
7)45217 +7 units over
6459 +4 eights over

The total remainder is evidently 39—that is, $(8 \times 4) + 7$.

(2) **By Italian Division—that is, writing down only the remainders in Long Division.**

We give the old and new methods side by side.

739)91856(124	739)91856(124
739	1795
1795	3176
1478	220
3176	
2956	
220	

N.B.—It will be advantageous to perform the operation of Subtraction by the method of **Complementary Addition**, in which, instead of *subtracting*, we ascertain how many must be *added* to one number to make it equal to another.

For example, take 4569 from 7384.

7384
4569
2815

Instead of saying 9 from 14, &c., we say (1) 9 and **5** are 14, put down **5** and carry 1 (the '1' being the 1 ten in 14); (2) 7 and **1** are 8, put down **1**; (3) 5 and **8** are 13, put down **8** and carry 1; (4) 5 and **2** are 7, put down **2**.

Applying this method to the preceding example in Long Division, we should begin, (1) once 9=9, 9 and **9** are 18, put down **9** and carry 1; (2) once 3=3, and 1 to carry=4, 4 and **7** are 11, put down **7** and carry 1; (3) once 7=7, and 1 to carry=8, 8 and **1** are 9, put down **1**.

Exercise 5.

Divide (*using factors*):

(1) 438574 by 20, 45 (2) 876549 by 27, 48 (3) 427564 by 72, 50
(4) 836095 by 49, 45 (5) 850701 by 55, 108 (6) 706589 by 72, 120

Divide (*using the contracted method*):

(7) 975638 by 34, 25, 46 (8) 869756 by 62, 83, 74
(9) 850973 by 69, 46, 58 (10) 657394 by 437, 645
(11) 982457 by 768, 235 (12) 420681 by 825, 734
(13) 574068 by 483, 297 (14) 8173645 by 1314, 3942
(15) 5810739 by 2438, 4876 (16) 6382970 by 9138, 18276

METHODS OF APPROXIMATION IN DECIMALS.

It often happens in arithmetic that numbers occur having many decimal places, while, for practical purposes, we need consider only one or two places. For example, the expression 136·7582 inches may occur. For many purposes, it would be sufficiently accurate to express this measurement by means of the whole number and only *two* decimal places. If we did this, we should not represent 136·7582 inches by 136·75 inches, but by 136·76 inches. In the former case the error

made would be ·0082 inch, or $\frac{82}{10000}$ of an inch, whereas in the latter the error made is only ·0018 inch, or $\frac{18}{10000}$ of an inch. Similarly, we should represent 8·7351 by 8·74, but 8·7349 by 8·73. In all these cases our answer is said to be 'correct to two decimal places;' or, to express the same fact in another way, 'correct to $\frac{1}{100}$.'

Rule.—When any figures are neglected in writing down a result, the last digit retained must be increased by 1 if the succeeding digit be 5 or greater than 5.

Limit of Error.—All measurements are made with a certain degree of accuracy. If we say the length of a room is 28 ft. 8 in., we understand that the length has been measured to the nearest inch, and that the actual length is greater than 28 ft. $7\frac{1}{2}$ in. and less than 28 ft. $8\frac{1}{2}$ in. The degree of accuracy here is $\frac{1}{2}$ in. If we say the length of a page of a book is 8·4 in., we understand that the length has been measured to the nearest $\frac{1}{10}$th of an inch, and that the actual length is greater than 8·35 in. and less than 8·45 in.—the degree of accuracy being ·05 in. In the same way, if we say a length is 8·46 in., we mean a length lying between 8·455 in. and 8·465 in.—the degree of accuracy being ·005 in.

Relative Error.—Suppose in estimating two lengths of 20 yd. and 100 yd. I make an error of 5 yd. in each case. The error in the first instance is much more serious than in the second, for it amounts to $\frac{1}{4}$ of the whole distance; whereas in the second case it is only $\frac{1}{20}$th of the whole distance. The fraction which expresses the relation of the error made to the true value is called the relative error.

$$\text{Relative Error} = \frac{\text{Error}}{\text{True Value}}.$$

It is evident that in measuring small quantities we must get a great degree of accuracy in our measurements if we are to avoid a large relative error.

Percentage Error.—This is obtained by expressing the Error as a percentage of the True Value.

ADDITION AND SUBTRACTION.

If we had several decimal numbers to add together, and wished to have the answer correct to a certain number of decimal places, it would only be necessary to extend the decimals to be added together one or, at most, two places beyond the number required.

For example, *Find the sum of* $16.\dot{3}2\dot{7}$, ·08, $26.\dot{8}6537\dot{}$, 28, $\frac{1}{6}$ *and* $\frac{2}{3}$, *correct to four places of decimals (that is, correct to* $\frac{1}{10000}$*).*

Extend the decimal places to six.

Thus $16.\dot{3}2\dot{7}$ $= 16.327327$
$\cdot 08 = \cdot 08$
$26.\dot{8}6537\dot{} = 26.865379$ (9 is a closer approximation than 8)
$28 = 28.$
$\frac{1}{6} = \cdot 2$
$\frac{2}{3} = \cdot 666667$ (7 is a closer approximation than 6)
Sum $= 72.139373$

Sum correct to four places of decimals $= 72.1394$.

In subtraction, as in addition, we may obtain the answer correct to a certain number of decimal places by extending the decimals one or two places beyond the number required.

For example, *Find the difference between* $17.\dot{8}6\dot{1}$ *and* $5\frac{3}{7}$, *correct to three places of decimals (that is, correct to* $\frac{1}{1000}$*).*

Extend the decimals to five places.

Thus, $17.\dot{8}6\dot{1} = 17.86186$
$5\frac{3}{7} = 5.42857$
Difference $= 12.43329$

Difference correct to three decimal places $= 12.433$.

NOTE.—When a decimal is correct to one place, it is, of course, correct to *one-tenth;* to two places, to *one-hundredth;* to three places, to *one-thousandth,* and so on.

EXERCISE 6.

Find the sums of the following:

(1) $\cdot 37\dot{0}\dot{2}$, $18.93\dot{6}$, 148, $4.5\dot{7}\dot{6}$, 84·00125, and ·375, correct to four decimal places.

(2) 1·875, $\cdot\dot{0}\dot{4}$, $\frac{2}{3}$, 40, $\cdot\dot{4}753\dot{2}$, and $\frac{1}{7}$, correct to three decimal places.

(3) $64.\dot{3}\dot{5}$, $\cdot 0\dot{2}8\dot{1}$, $5.\dot{1}4307\dot{}$, 96, $\frac{5}{8}$, and $\frac{11}{13}$, correct to five decimal places.

(4) $\cdot 0287614$, $62\cdot\dot{5}$, $19\cdot4\dot{8}\dot{7}\frac{5}{7}$, $\cdot 0056$, $\frac{13}{19}$, and $701\cdot58243$, correct to two decimal places.

(5) $30\cdot4\dot{8}$, $11\cdot\dot{9}1\dot{5}$, $\cdot\dot{0}0\dot{2}$, 450, $\frac{3}{4}$, $27\frac{4}{7}$, and $1\frac{1}{2}$, correct to three decimal places.

(6) $\cdot\dot{7}2\dot{8}$, $25\frac{3}{10}$, $\frac{4}{7}$ of $\frac{14}{15}$, $12\frac{1}{22}\frac{1}{2}$, $\cdot\dot{4}$, $100\cdot\dot{3}0\dot{6}$, and $\frac{2}{11}$ of $8\frac{1}{3}$, correct to six decimal places.

Find

(7) (*a*) $17\cdot3\dot{4} - 12\cdot5$; (*b*) $\cdot\dot{2}8\dot{3} - \cdot01\dot{6}$, correct to four decimal places.

(8) (*a*) $\frac{5}{6} - \frac{2}{3}$; (*b*) $18 - \cdot2\dot{0}3\dot{4}$, correct to five decimal places.

(9) (*a*) $1\cdot7\dot{0}93\dot{2} - \frac{7}{11}$; (*b*) $\frac{5}{7} - \frac{1}{17}$, correct to six decimal places.

(10) (*a*) $145\cdot3\dot{4} - 29\cdot76\dot{4}$; (*b*) $20\cdot2 - 19\cdot\dot{8}20\dot{1}$, correct to three decimal places.

(11) (*a*) $53\frac{3}{4} - \frac{13}{19}$; (*b*) $\cdot\dot{1} - \cdot0\dot{1}$, correct to two decimal places.

(12) $100 - 10\cdot\dot{3} - 46\cdot2\dot{8} - \frac{5}{9}$, correct to three decimal places.

Reason for adopting Approximate Methods.

In using quantities which are the results of measurements, we must be careful not to assume a greater accuracy in our results than the data warrant. For example, if the page of a book is 8·76 in. long and 5·23 in. broad, what is its area? To get the area we multiply the length by the breadth, and we obtain 45·8148 sq. in. Let us now ascertain how far this result is reliable.

The length of the page lies between 8·755 in. and 8·765 in.,
the breadth " " 5·225 in. " 5·235 in.,
the area " " 45·744875 sq. in. and
45·884775 sq. in.,

so that the answer we obtained was approximately accurate only to the first decimal place.

The degree of accuracy of such a result could be obtained thus. The limit of error in both measurements is ·005 in.; therefore the area might be as large as $(8\cdot76 + \cdot005)(5\cdot23 + \cdot005)$

$= 8\cdot76 \times 5\cdot23 + 8\cdot76 \times \cdot005 + 5\cdot23 \times \cdot005 + \cdot005 \times \cdot005$

$= 8\cdot76 \times 5\cdot23 + \cdot0438 + \cdot02615 + \cdot000025$ [the last term is so small that it may be neglected]

$= 8{\cdot}76 \times 5{\cdot}23 + {\cdot}06995$

$= 45{\cdot}8148 + {\cdot}06995.$

Similarly, the area might be as small as

$8{\cdot}76 \times 5{\cdot}23 - {\cdot}06995$

$= 45{\cdot}8148 - {\cdot}06995.$

So that all we are entitled to conclude about the area is that it is within ·07 (·06995) of 45·81. The second, third, and fourth decimal figures in our first result, then, are absolutely valueless.

Had the measurements been given as 8·7 in. by 5·23 in., the limit of error in the area would have been $8{\cdot}7 \times {\cdot}005 + 5{\cdot}23 \times {\cdot}05 = {\cdot}3$, so that in this case even the first decimal figure is liable to serious error.

The limit of error in a product may be found quickly by the following rule: Divide each factor by 2, then move the decimal point in each result as many places to the left as there are decimal figures in the other factor, add, and the sum gives the possible error.

It is thus apparent that it is often useless to obtain all the figures in a multiplication sum by the ordinary methods, and in the following sections we proceed to show how the work may be contracted.

MULTIPLICATION.

Suppose we wish to multiply 367 by 839. In doing this, we may multiply either by the 9 or by the 8 first, whichever we prefer. The working by the two methods would appear thus:

(*a*)	(*b*)
367	367
839	839
3303	2936
1101	1101
2936	3303
307913	307913

In contracted multiplication of decimals we shall employ the latter method.

There are several ways of working contracted multiplication of decimals. The method explained here is perhaps the one most easily understood, and the one most readily put into practice.

Before giving the rule, however, we must draw attention to two preliminary considerations.

(1) Since $\cdot 2 \times \cdot 4$ gives $\cdot 08$, we see that a figure in the first decimal place multiplied by a figure in the first decimal place gives a figure in the second decimal place of the product; or, more briefly, a first decimal place multiplied by a first decimal place gives a second decimal place. Since $\cdot 02 \times \cdot 4$ gives $\cdot 008$, we see that a second decimal place multiplied by a first decimal place gives a third decimal place.

Similarly, a third, fourth, fifth, ... decimal place multiplied by a first decimal place gives a fourth, fifth, sixth, ... decimal place; a third, fourth, fifth, ... decimal place multiplied by a second decimal place gives a fifth, sixth, seventh, ... decimal place; and so on.

In like manner, we could easily show that a figure in the first, second, third, ... decimal place multiplied by a figure in the units' place will give a figure in the first, second, third, ... place in the product.

$$\cdot 2 \times 4 = \cdot 8.$$

(2) Suppose it were required to multiply 89 by 40. If, instead of the factors 89 and 40, we take as factors 890 and 4, the result will be the same, for the first factor has been made ten times larger than it was, and the second factor ten times less.

$$89 \times 40 = 3560; \quad 890 \times 4 = 3560.$$

Similarly, if it were required to multiply 3619·4 by ·082, the result would be the same as if we multiplied 36·194 by 8·2, for the first factor has been made a hundred times

less than it was, and the second factor a hundred times larger.

$3619{\cdot}4 \times {\cdot}082 = 296{\cdot}7908\,;\ 36{\cdot}194 \times 8{\cdot}2 = 296{\cdot}7908.$

Hence, we can always arrange that the multiplier shall contain only one figure in the 'whole number,' by moving the decimal point the requisite number of places either to the right or to the left. A removal of the decimal point the *same* number of places, but in the *opposite* direction, must be made in the multiplicand.

We are now in a position to explain the method adopted for contracted multiplication of decimals.

Suppose we wish to multiply 6081·91358 by ·025157537, correct to three places of decimals.

(1) Make the multiplier contain only one figure in the 'whole number,' by moving the decimal point two places to the right—that is, make it 2·5157537. Correspondingly, move the decimal point of the multiplicand two places to the left, making it 60·8191358.

(2) Write down the new multiplier and the new multiplicand with their decimal points exactly under each other. To ensure that our answer is correct to *three* places of decimals, we work the question as if we had to obtain *four* decimal places. To do this we cut off all the decimal places beyond four in both multiplier and multiplicand.

60·8191	358	Rough check (see paragraph 7),
2·5157	537	$6082 \times {\cdot}025 = 152{\cdot}05$.
121·6383		beginning, 'twice 1 and 1 to carry.'
30·4096		5 times 9 " 1 "
·6082		once 1 " 1 "
·3041		5 times 8 " 1 "
·0426		7 " 0 " 6 "
·0030		5 " 6 " 0 "
2		3 " 0 " 2 "
153·0060		

$= 153{\cdot}006$, correct to three places.

(3) We multiply by the units' figure (the 2) first. As we

are working to obtain only four decimal places in our answer, we need only begin to multiply at the fourth decimal place in the multiplicand (the 1), for the fourth decimal place multiplied by the units' figure will bring a fourth decimal place in the answer. We must, however, take care to add in anything there would have been to carry from the multiplication of the figures neglected—namely, 358. (In this case we add 1, because $358 \times 2 = 716$, and 7 is nearer 10 than 0.)

(4) We next multiply by the figure in the first decimal place (the 5). As a third decimal place multiplied by a first decimal place gives a fourth decimal place, we need only begin to multiply at the third decimal place (the 9), taking care to add in the carrying figure (1 in this case), for the reason given above.

N.B.—The first figure of every partial product is, of course, placed in the fourth decimal place.

(5) We next multiply by the figure in the second decimal place (the 1). As a second decimal place multiplied by a second decimal place gives a fourth decimal place, we need only begin to multiply at the second decimal place (the 1), taking care to add in the carrying figure (in this case, 1).

(6) For the next line in the product we multiply by the figure in the third decimal place, and we need only begin to multiply at the first decimal place.

In other words, as we take a step to the right (as it were) in the multiplier, we take a corresponding step to the left in the multiplicand.

We continue in this way until we have exhausted all the figures in the multiplier, and then proceed to add. After the addition we strike out the fourth decimal place, correcting the third, if necessary.

(7) The student should always check his result by means of a rough check. Take for the factors approximate values which are easily multiplied, and compare approximate result with the result obtained. This will often prevent a student

from giving in a result 10 or 100 times as large as it should be.

The working of two more examples may be given.

(*a*) *Multiply* ·18234 *by* ·00731, *retaining four decimal places in the product.*

$$\cdot 18234 \times \cdot 00731 = \cdot 00018234 \times 7 \cdot 31.$$

·00018 | 234	Rough check, $\cdot 18 \times \cdot 007 = \cdot 00126$.
7·31	
·00128	beginning, '7 times 8 and 2 to carry.'
·00005	" 3 " 1 " 2 "
·00133	

$= \cdot 0013$, correct to four places of decimals.

(*b*) *Multiply* 56·3135 *by* 24·018, *correct to the units' figure.*

$$56 \cdot 3135 \times 24 \cdot 018 = 563 \cdot 135 \times 2 \cdot 4018.$$

563·1 | 35	Rough check, $56 \times 24 = 1344$.
2·4 | 018	
1126·3	beginning, 'twice 1 and 1 to carry.'
225·3	" 4 times 3 " 1 "
·6	" once 5 " 1 "
·4	" 8 times 0 " 4 "
1352·6	

$= 1353$, correct to the units' place.

Exercise 7.

Multiply:

(1) 17·3256	*by* 8·14205345,	*retaining*	3	*decimals.*
(2) 608·191358	" ·0251575,	"	3	"
(3) 2·81809781	" ·511329,	"	4	"
(4) ·05656	" 1145·269007,	"	2	"
(5) 12·22361814	" 2·11529768,	"	3	"
(6) ·26927	" ·0015051,	"	4	"
(7) 1·92648	" ·007537,	"	4	"
(8) 3·672	" ·477108,	"	3	"
(9) 32·01	" 1·0246401,	"	4	"
(10) ·01637	" ·2679769,	"	4	"
(11) 20·985561	" 4·581,	"	2	"
(12) 7·829	" 6129·3241,	"	2	"
(13) 4·5307	" ·0002518376,	"	6	"
(14) 56·388147	" 429·7846,	"	0	"
(15) 853·2479	" 48·125,	"	0	"

(16) 17·32456	*by* 4·2937,	*retaining*	2	*decimals.*
(17) 8524·3075	" ·5367,	"	1	"
(18) 73·56148	" 28·295,	"	2	"
(19) ·54870321	" ·71314,	"	4	"
(20) 43·568401	" ·0032175,	"	2	"
(21) ·80512347	" ·00312548,	"	5	"
(22) 8·571326	" 27·04753,	"	2	"
(23) 98·720486	" 542·1723,	"	0	"
(24) 5834·491	" 78·6923,	"	0	"

DIVISION.

$80 \div 20 = 4$, $800 \div 200 = 4$, and $8 \div 2 = 4$. Therefore $80 \div 20 = 800 \div 200 = 8 \div 2$. In other words, if we multiply or divide both dividend and divisor by the same number, the quotient is not altered.

The same rule holds in the case of decimals. $\cdot 8 \div \cdot 4 = 2$; $\cdot 08 \div \cdot 04 = 2$; and $8 \div 4 = 2$. Hence we can move the decimal points of *both* divisor and dividend without altering the quotient, provided we move them the same number of places and in the same direction.

Having considered this preliminary principle, we are now in a position to give a method for contracted division of decimals.

Suppose we wish to divide 932·58 by 24·8153, correct to three decimal places.

To ensure that our answer is correct to *three* places of decimals, we work the question as if we had to obtain *four* decimal places.

(1) Make the divisor contain only one figure in the 'whole number,' and alter the dividend accordingly, adding 0's if necessary. (In this case move the decimal point one place to the left—that is, make it 2·48153. Correspondingly, move the decimal point of the dividend one place to the left, making it 93·258.)

(2) Determine how many figures there will be in the answer. (We know there are to be four figures in the

decimal part of the answer, and we can see by inspection that the 'whole number' will contain two figures, making six in all.)

(3) Divide as usual until the number of figures remaining to be found in the quotient is one less than the number of figures in the divisor. (In this case there are six figures in the divisor, and, as we have seen, there are to be six in the answer. Therefore, we divide as usual only once.)

(4) When this point is reached, instead of 'bringing down the next figure' of the dividend, cut off the right-hand figure of the divisor. (In this case cut off the 3.)

(5) Divide by this new divisor, *but remember to use the rejected figure to find out what must be 'carried.'*

(6) Proceed in this way, cutting off the figures one by one, until the required number of decimal places is obtained.

2,·4,8,1,5,3)93·2580(37·5808

74 4459 Rough check, $900 \div 25 = 36$.

18 8121

17 3707 beginning, '7 times 5 and 2 to carry.'

1 4414

1 2408 " 5 " 1 " 3 "

2006

1985 " 8 " 8 " 1 "

21

20 " 8 " 2 " 4 "

1

The quotient (37·5808) thus obtained, when corrected to three places of decimals, = 37·581.

By the shortened way of working division previously given, the working would appear thus :

2,·4,8,1,5,3)93·2580(37·5808

18 8121

1 4414

2006

21

1

N.B.—If there are originally more figures in the divisor than there are to be in the quotient, we may at once strike out the unnecessary figures, and proceed as above. For example, if the divisor contains eight figures, and the quotient is to contain four in all, we may at once strike out three figures from the divisor, thus leaving five—that is, one more than the quotient is to contain. (We, of course, strike out another figure to get the first actual divisor.)

Proof of the Rule.—Multiply the original divisor, 24·8153, by the actual quotient thus found, obtaining an answer correct to two places of decimals (the number of decimal places contained in the original dividend, 932·58).

By the method previously given, the working appears as follows:

```
248·153 |
  3·758 | 08
---------
744·459
173·707
 12·408
  1·985
   ·020
---------
932·579 |
```

=932·58, correct to two places of decimals.

If we examine each line of the above we shall find that each partial product corresponds to a product obtained in working the exercise in division. Thus, in both the division and the multiplication exercises we have to multiply 248153 by 3, 24815 by 7, 2481 by 5, 248 by 8, and so on.

Thus, as we were justified in cutting off or neglecting a figure at a time in multiplying, so we are at liberty to cut off a figure at a time in dividing.

Exercise 8.

Divide

(1) 12·2574 *by* 5·3568 *correct to* 3 *decimal places.*
(2) 7·143562 " 1·28143 " 4 " "
(3) ·827134 " 10·567 " 4 " "
(4) ·04273 " 58·19 " 6 " "
(5) ·2846537 " ·32856 " 3 " "
(6) 73·12 " ·5284679 " 2 " "

(7) 49·023	*by* 764·51	*correct to*	5	*decimal places.*	
(8) 1·85	" 203·29	"	6	"	"
(9) 35·287946	" 4·562374	"	2	"	"
(10) 285·46901364	" 73·21689125	"	2	"	"
(11) 4·5823716	" 2·3984127	"	3	"	"
(12) ·547302954	" 8·23401763	"	4	"	"
(13) 23·516784	" 8·379248	"	3	"	"
(14) 1·37206843	" ·829716348	"	3	"	"
(15) 57·6943	" ·03256914	"	1	"	"
(16) 9·0147652	" ·0071328	"	2	"	"
(17) ·8427693	" 15·279108	"	4	"	"
(18) ·03916257	" 234·60514	"	6	"	"
(19) ·00523819	" 4863·75	"	9	"	"
(20) ·12345678	" 9·8765432	"	4	"	"
(21) 7·3867914	" 82·156349	"	5	"	"
(22) ·518273849	" 467·285743	"	6	"	"
(23) ·256843917	" 182·56437	"	6	"	"
(24) 735·28	" ·01523764	"	units.		
(25) 1079·5624	" ·0037625	"	"		
(26) 2500	" ·015625	"	"		

Significant Figures.

Significant figures are the figures which express a given quantity in terms of the unit employed. For example, we may say the circumference of the earth is 25,000 miles, meaning that the distance is 25 thousand miles measured to the nearest thousand miles. Here the figures 2, 5 are significant, while the ciphers simply indicate the unit we are employing, and are said to be non-significant. If we mean that the distance has been measured to the nearest hundred miles, then the 2, 5, 0 are significant, and the other two ciphers are non-significant.

Again, '1 in. = 2·5 cm.' means that we have measured 1 in. to the nearest tenth of a centimetre, and both figures are significant.

Similarly, '1 sq. cm. = ·155 sq. in.' means that our measurement is correct to the nearest thousandth of a square inch, and all the figures are significant.

In '1 sq. mm. = ·00155 sq. in. = $\frac{155}{100000}$ sq. in.' our measurement is correct to the nearest hundred-thousandth of a square inch, and the 1, 5, 5 alone are significant. The ciphers simply indicate the unit we are employing.

If we say that 1 sq. mm. = ·001550 sq. in. we indicate that our measurement is correct to the nearest millionth of a square inch, and here the cipher at the end is significant, while the ciphers at the beginning are non-significant.

Summary.—Ciphers at the beginning of a pure decimal are non-significant.

Ciphers at the end of a decimal are significant.

Ciphers at the end of a whole number may, or may not, be significant according to the degree of accuracy of our measurement.

Exercise 9.

I. *Point out what figures are significant in the following measurements, giving your reason in each case:*

(1) Distance of Earth from Sun = 93,000,000 miles, to the nearest million miles.
(2) Diameter of Earth = 3970 miles, to nearest 10 miles.
(3) Distance London to Edinburgh = 400 miles, to nearest mile.
(4) 1 cm. = ·3937 in.
(5) 1 acre = ·40 hectares.
(6) 1 sq. cm. = ·00012 sq. yd.
(7) 1 metre = 1·094 yd.
(8) 1 litre = ·2201 gal.
(9) 1 kilogram = 2·20 lb.

II. *Use contracted methods to find the first 4 significant figures in the following:*

(1) ·06112 × ·9214
(2) 495·3 × 26·87
(3) 4359 × ·01985
(4) ·07835 × ·9864
(5) 594·6 × 38·29
(6) 5649 × ·02873
(7) ·09847 ÷ 85·73
(8) 59·83 ÷ 8·472
(9) 28·73 ÷ 16·84
(10) 4·652 ÷ 685·9
(11) 65·49 ÷ 7·835
(12) ·06485 ÷ 53·84
(13) 39·41 ÷ 28·35
(14) 4·683 ÷ 782·5
(15) 48·59 × 3·875 ÷ 965·4
(16) 56·73 × 4·382 ÷ 865·4

III. *If the figures given in* II. (1–16) *represent the results of measurements, show in each case, by finding the limit of error, how far your answer is reliable.* [Refer to page 25.]

NOTE.—The student familiar with the elementary algebraical identity $(a+b)(a-b)=a^2-b^2$ will have no difficulty in forming a rule for the limit of error in division. Note that the error is greatest when one factor is increased and the other diminished. In II. (8) above

$\frac{59{\cdot}83}{8{\cdot}472}$ may represent $\frac{59{\cdot}83+{\cdot}005}{8{\cdot}472-{\cdot}0005}=\frac{(59{\cdot}83+{\cdot}005)(8{\cdot}472+{\cdot}0005)}{(8{\cdot}472)^2-({\cdot}0005)^2}$

The limit of error in numerator is ·072 (see page 26), and this has to be divided by $(8{\cdot}472^2)$. It will be sufficient to take the approximate value of this, say 8^2, and dividing ·072 by 64, we get the limit of error = ·001.

SHORT METHODS IN THE DECIMALISATION OF MONEY AND WEIGHTS AND MEASURES.

I. **General Method.**—Begin at the right-hand term and express it as a decimal of the next highest denomination. Repeat this as often as necessary.

EXAMPLES.—(1) *Express* 10*s.* 9¾*d. as the decimal of* £1.

10s. 9¾d. = 10s. 9·75d. = 10·8125s. = £·540625.

The 3 farthings divided by 4 = ·75 pence; the 9·75 pence divided by 12 = ·8125 shillings; the 10·8125 shillings divided by 20 = £·540625.

(2) *Express* 3 *lb.* 8 *oz. as the decimal of* 1 *cwt.*

3 lb. 8 oz. = 3·5 lb.
= ·03125 cwt.

$$112\left\{\begin{array}{r|l} 7 & 3{\cdot}5 \\ 4 & {\cdot}5 \\ 4 & {\cdot}125 \end{array}\right.$$
·03125

(3) *Express* 3 *qt.* 1¾ *pt. as the decimal of* 1 *gallon.*

3 qt. 1¾ pt. = 3 qt. 1·75 pt.
= 3·875 qt.
= ·96875 gal.

II. To reduce shillings, pence, and farthings to the decimal of £1, correct to three places.

For example, reduce 17*s.* 6¾*d.* to the decimal of £1, correct to three places.

(1) *Pence and Farthings.*—Reduce to farthings. A farthing $= £\frac{1}{960}$; that is, it is rather larger than $£\frac{1}{1000}$, or £·001. Therefore, multiply ·001 by the number of farthings, first increasing them by 1 if they number 12 or more, and by 2 if they number 36 or more. For example, $6\frac{3}{4}$d. = 27 farthings = £·028. (The addition of 1 or 2 to the number of farthings corrects the error made in taking a farthing as exactly £·001.)

(2) *Shillings.*—1s. = £·05; therefore, multiply the number of shillings (17) by 5, and write the product in the first and second decimal places = £·85. (This is, of course, equivalent to multiplying the shillings by ·05.)

(3) Add the two results together; in this case £·85 + £·028 = £·878; therefore, 17s. $6\frac{3}{4}$d. = £·878.

Short Statement of Rule.—The method here explained might be given briefly as follows: Reduce the pence and farthings to farthings, and increase them by 1 if they number 12 or more, and by 2 if they number 36 or more. Place the result in the third decimal place. Multiply the number of shillings by 5, add the figure carried from the third decimal place, if any, and write the result in the first and second decimal places.

For example, 13s. $11\frac{1}{2}$d. = £·698.

III. To reduce a decimal of £1, correct to three places, to shillings, pence, and the nearest farthing.

For example, bring £·639 to shillings, pence, and farthings.

N.B.—If necessary, *make* the decimal correct to three places. (For example, £·58571 = £·586, correct to three places.)

(1) *Shillings.*—Reversing the above rule, divide the first and second decimal places by 5, and the quotient is the number of shillings required; for example, 63 ÷ 5 = 12 and 3 over.

(2) *Pence and Farthings.*—The *whole* remainder, diminished by 1 if larger than 12, and by 2 if larger than 36, will be farthings. In this case 39 is the whole remainder, and, therefore, 37 is the number of farthings.

Therefore, £·639 = 12s. $9\frac{1}{4}$d.

IV. To reduce shillings, pence, and farthings to the decimal of £1, at sight.

For example, reduce 17s. 2¼d. to the decimal of £1.

(1) *Shillings.*—1s. = £·05; therefore multiply the number of shillings (17) by 5, and write the product in the first and second decimal places—£·85.

(2) *Pence and Farthings.*—Reduce to farthings.

1s. = £·05;

∴ 1d. = £·004$\frac{1}{6}$;

∴ ¼d. = £·001$\frac{1}{24}$;

therefore multiply the number of farthings (9) by ·001$\frac{1}{24}$, and write the product in the third decimal place—£·009$\frac{9}{24}$.

∴ 17s. 2¼d. = £·859$\frac{9}{24}$.

The decimal may be extended by expressing the 9 farthings as pence (2·25d.) and dividing by 6. This is equivalent to dividing the farthings by 24.

∴ 17s. 2¼d. = £·859375.

Note that if the pence be 6d. or more than 6d. the terminal fraction is 1 or greater than 1, and therefore we must add 1 to the third decimal place and extend the remaining fraction as before.

For example, 8¼d. = 33 farthings

= £·033$\frac{33}{24}$

= £·034$\frac{9}{24}$ (9 far. = 2·25d.)

= £·034375.

Rule.—Reduce the pence and farthings to farthings; add 1 if the pence be 6d. or greater than 6d.; place the result in the third decimal place.

Multiply the shillings by 5, adding the figure carried from the third decimal place, if any, and place the result in the first and second decimal places.

Divide the pence and farthings, or the excess of pence and farthings over 6d., expressed in pence, by 6, and place the result in the fourth and succeeding decimal places.

For example, £3, 14s. 8¼d. = £3·735416.

Mental Working.—8¼d. = 34 far., add 1 since the pence exceed 6d., 35, place 5 in third decimal place and carry 3; 14 × 5 = 70, add 3 carried, 73, and place in the first and second decimal places; divide 2½d. (2·5) by 6 and place the result in the fourth and succeeding decimal places.

Exercise 10.

Reduce:

(1) 18s. 3d. ; 15s. 6¾d. ; 8½d. ; 6s. 1½d. to the decimal of £1.

(2) 4s. 8d. to the decimal of a guinea ; 2s. 4½d. to the decimal of a crown ; 3 qr. 14 lb. to the decimal of a cwt. ; 2 fur. 6 po. to the decimal of a mile.

(3) 1 ro. 19 po. to the decimal of 1 acre ; 12 hr. 36 min. to the decimal of a week ; 11 yd. to the decimal of a mile ; 1½d. to the decimal of a shilling.

Reduce to decimals of £1, correct to three places:

(4) 13s. 7¼d. ; 10s. 4½d. ; 5s. 6½d. ; 11s. 2¼d.

(5) 12s. 9¾d. ; 17s. 5½d. ; 7s. 3½d. ; 18s. 4¼d.

(6) 3s. 11½d. ; 14s. 1½d. ; 17s. 9½d. ; 10s. 11¾d.

(7) 8s. 2½d. ; 13s. 1¾d. ; 15s. 9¼d. ; £3, 14s. 0¼d.

(8) 4s. 5¾d. ; £8, 19s. 0¾d. ; 17s. 0¼d. ; 9s. 3¾d.

(9) 1s. 10¼d. ; 12s. 11¼d. ; 6¾d. ; 10½d.

(10) 5d. ; 1s. 6½d. ; £10, 1s. 1⅞d. ; 1d.

(11) ¼d. ; 1s. 2¾d. ; 19s. 11¾d. ; 6s. 10½d.

Find to the nearest farthing the values of the following decimals of £1, first correcting to three places where necessary:

(12) ·068 ; ·122 ; ·006 ; ·307 ; ·675 ; ·871.

(13) ·703125 ; ·76875 ; ·10625 ; ·16875 ; ·2.

(14) 6·353125 ; ·3875 ; ·715625 ; ·275 ; ·75.

(15) ·965625 ; ·25 ; ·89375 ; 8·95 ; ·88125 ; ·3̇.

(16) 2·7 ; ·525 ; 3·06875 ; ·0125 ; ·008$\dot{3}$; ·5.

Use the method given in paragraph IV. to work examples (4) to (11), expressing each sum correctly as a decimal of £1. The answers are numbered (17) to (24).

SHORT METHODS USED IN THE CALCULATION OF PRICES.

Note.—*Most of the examples in this section should be worked mentally.*

To find the price of a dozen. Consider the number of pence in the price as shillings, ½d. as 6d., ¼d. as 3d., and ¾d. as 9d.

For example, **12 articles at 1s. 7¾d. each = 19s. 9d.**

This rule depends upon the facts that there are 12 pence in a shilling, 12 halfpennies in 6d., and so on.

(1) Find the price of one dozen articles at 7½d., 4¼d., 8¾d., 1s. 2½d., and 2s. 4¼d. each article.

To find the price of any number of dozens. Find the price of one dozen by the above rule, and multiply by the number of dozens.

(2) Find the price of 2, 3, 7, and 11 dozen articles at 3¾d., 8½d., 1s. 3¼d., and 2s. 6½d. each article.

To find the price of a gross. A gross = a dozen dozens, therefore apply the rule for finding the price of a dozen; *or*, make table showing price of 144 articles at 1s., 1d., 1f., thus:

	£	s.	d.
s.	7	4	0
d.	0	12	0
f.	0	3	0

Price of 144 at 3s. 4½d.

	=	£	s.	d.
		21	12	0
		2	8	0
←		0	6	0
	=	£24	6	0

(3) Find the price of a gross of articles at 8¼d., 10¾d., 1s. 4½d., and 2s. 7½d. each article.

To find the price of any number of gross. Find the price of one gross, and multiply it by the number of gross; *or*, if more convenient, multiply the price by that number, and treat the result by the rule for one gross.

(4) Find the price of 2, 3, 5, and 7 gross of articles at 2¼d., 6½d., 1s. 2d., and 2s. 3¾d. each article.

To find the price of a score. Consider the number of shillings in the price as pounds, and the pence as twelfths of a pound.

For example, 20 articles at £3, 5s. 8d. each = £65$\frac{8}{12}$ = £65$\frac{2}{3}$ = £65, 13s. 4d.

This rule depends upon the fact that there are 20 shillings in £1.

(5) Find the price of a score of articles at £1, 2s. 3d., £4, 5s. 7d., 4s. 6½d., and £1, 3s. 2¼d. each article.

To find the price of any number of scores. Find the price of one score, and multiply it by the number of scores; *or* (if more convenient) multiply the price by that number, and treat the result by the rule for one score.

(6) Find the price of 2, 6, 7, and 11 scores of articles at 7s. 2d., £1, 2s. 4d., 1s. 8½d., and 3s. 4¼d. each article.

To find the price of 48 articles. Consider the number of farthings in the price as shillings.

For example, 48 articles at **1s. 9½d.** each = **86s. = £4, 6s.**

This rule depends upon the fact that there are 48 farthings in a shilling.

(7) Find the price of 48 articles at 2¼d., 8½d., 1s. 4¾d., and 2s. 4¼d. each.

To find the price of 100 articles. Consider each shilling in the price as £5, and add as many florins and pence as there are farthings in the pence and farthings of the price.

For example, 100 articles at **17s. 9¾d.** each = **£85 + 39 florins + 39d. = £85 + £3, 18s. + 3s. 3d. = £89, 1s. 3d.**

This rule depends upon the facts that £5 = 100 shillings, and that 100 farthings = 2s. 1d. (a florin and a penny).

(8) Find the price of 100 articles at 8s. 2½d., 11s. 2¼d., 1s. 2¾d., and 13s. 7¼d. each.

To find the price of 240 articles. Consider the pence as pounds, ¼d. as 5s., ½d. as 10s., and ¾d. as 15s.

For example, 240 articles at **1s. 10¾d.** each = **£22, 15s.**

The reason for this rule is that 240d. = £1.

(9) Find the price of 240 articles at 4½d., 11¾d., 1s. 5¼d., and 2s. 2½d. each.

To find the price of 480 articles. Consider the halfpence as pounds, and ¼d. as 10s.

For example, 480 articles at **1s. 4¾d.** each = **£33, 10s.**

(10) Find the price of 480 articles at 3¼d., 10½d., 1s. 3¾d., and 2s. 6½d. each.

To find the price of 960 articles. Consider the farthings as pounds.

For example, 960 articles at **7¾d.** each = **£31.**

(11) Find the price of 960 articles at 2¼d., 8¾d., 1s. 7½d., and 3s. 5¼d. each.

To find the price of 10, 100, 1000, &c., articles. Express the price as the decimal of £1, and move the decimal point 1, 2, or 3 places to the right.

For example, 1000 articles at 15s. 5¼d. each = £771·875 = £771, 17s. 6d.

(12) Find the price of 10, 100, and 1000 articles at 1s. 6½d., 6s. 10½d., 6¾d., and 18s. 4½d. each.

To find the price of 365. As 365 pence = 240d. + 120d. + 5d. = £1 + 10s. + 5d., the following rule may be given: Reckon as many sovereigns, as many half-sovereigns, and five times as many pence as there are pence in the price of one article.

For example, 365 articles at 7s. 5d. each = £89 + £44, 10s. + £1, 17s. 1d. = £135, 7s. 1d.

(13) Find the price of 365 articles at 4d., 8½d., 1s. 2½d., and 3s. 6½d. each.

To find the price of a large number of articles at so many pence each. For example, 112 lb. at 10½d. per lb. This evidently = 10½d. × 112, which is equal to 112d. × 10½ = 9s. 4d. × 10½ = £4, 18s. Hence we may give the rule, Consider the number of articles as pence, and multiply by the number of pence in the price.

Similarly, to find the price of a large number of articles at so many shillings each, we may consider the number of articles as shillings, and multiply by the number of shillings in the price.

(14) Find the price of 217, 843, 1296, and 453 articles at 2¾d., 7½d., 9½d., and 11½d. each.

To find the price of a number of articles, where the cost of each is an aliquot part of £1, or of a shilling. Divide the number of articles by the denominator of the fraction which expresses the aliquot part, and the answer will be pounds, or shillings, as the case may be.

For example, 358 articles at 3s. 4d. each = £(358 ÷ 6) = £59, 13s. 4d.; 269 articles at 1½d. each = (269 ÷ 8)s. = 33s. 7½d. = £1, 13s. 7½d.

The student should be on the watch for opportunities of applying the above rules to numbers or prices differing but little from those given. For example, (1) to find the cost of 86 articles, find the price of 7 dozen, and add the price of 2 articles; (2) to find the price of 139 articles, find the price of 7 score, and subtract the price of 1 article; (3) to find the cost of so many articles at 11½d. each, find the cost at 1s. each, and subtract the cost at ½d. each.

(15) Find the price of 743, 218, 29, and 876 articles at 6s. 8d., 2s. 6d., 1s. 8d., and 1½d. each.

Method of Nine Multiples.—Suppose a merchant is selling some commodity at a fixed price, say 3s. 7½d. He may calculate his prices quickly as follows: Express the price (3s. 7½d.) as a decimal of £1, and make a table of the products obtained by multiplying this price by the numbers 1 to 9.

	Table.	
1	3s. 7½d. = £ ·18125	Price of 40 = £7·25
2	·3625	" " 9 = 1·63125
3	·54375	" " 49 = £8·88125 = £8, 17s. 7½d.
4	·725	Price of 30 = £5·4375
5	·90625	" " 7 = 1·26875
6	1·0875	" " 37 = £6·70625 = £6, 14s. 1½d.
7	1·26875	Price of 700 = £126·875
8	1·45	" " 60 = 10·875
9	1·63125	" " 5 = ·90625
		" " 765 = £138·65625 = £138, 13s. 1½d.

In finding the price of 49 articles, we obtain the price of 4 from the table, multiply by 10, that is, move the decimal point one place to the right, and thus obtain the price of 40. We then obtain the price of 9 from the table and add.

This method is specially useful where varying quantities are sold at a fixed price. It is also very convenient for expressing foreign money in £, s. d. For example, If a rupee = 1s. 4½d., find the value of 29, 475, and 1000 rupees in £, s. d.

	Table.	
1	1s. 4½d. = ·06875	20 rupees = £1·375
2	·1375	9 " = ·61875
3	·20625	29 " = £1·99375 = £1, 19s. 10½d.
4	·275	400 rupees = £27·5
5	·34375	70 " = 4·8125
6	·4125	5 " = ·34375
7	·48125	475 " = £32·65625 = £32, 13s. 1½d.
8	·55	1000 rupees = £68·75 = £68, 15s.
9	·61875	

Exercise 11. (*Mental.*)

Find the cost of the following numbers of articles, at the given price each:

(1) (*a*) 12, (*b*) 84, at 1s. 9$\frac{3}{4}$d.
(2) (*a*) 48, (*b*) 240, at 8$\frac{1}{4}$d.
(3) (*a*) 20, (*b*) 80, " 3s. 7$\frac{1}{2}$d.
(4) (*a*) 60, (*b*) 97, " 1s. 1$\frac{1}{4}$d.
(5) (*a*) 365, (*b*) 120, " 2s. 9d.
(6) (*a*) 115, (*b*) 81, " 9d.
(7) (*a*) 278, (*b*) 315, " 2s. 6d.
(8) (*a*) 180, (*b*) 100, " 2s. 9d.
(9) (*a*) 240, (*b*) 96, " 8$\frac{3}{4}$d.
(10) (*a*) 437, (*b*) 100, " 4d.
(11) (*a*) 61, (*b*) 83, " 7$\frac{1}{2}$d.
(12) (*a*) 77, (*b*) 126, " 11$\frac{1}{2}$d.
(13) (*a*) 160, (*b*) 95, " 5$\frac{1}{2}$d.
(14) (*a*) 135, (*b*) 71, " 8d.
(15) (*a*) 79, (*b*) 818, " 6s. 8d.
(16) (*a*) 365, (*b*) 36, " 1s. 10d.
(17) (*a*) 84, (*b*) 100, " 10$\frac{1}{2}$d.
(18) (*a*) 141, (*b*) 13, " 1s. 9d.
(19) (*a*) 212, (*b*) 179, " 19s. 11d.
(20) (*a*) 568, (*b*) 361, " 1s. 8d.
(21) (*a*) 100, (*b*) 365, " 3s. 5d.
(22) (*a*) 119, (*b*) 100, " 6$\frac{1}{4}$d.
(23) (*a*) 127, (*b*) 367, " 8$\frac{1}{2}$d.
(24) (*a*) 99, (*b*) 181, " 5s. 5d.
(25) (*a*) 171, (*b*) 83, " 9$\frac{1}{4}$d.
(26) (*a*) 87, (*b*) 113, " 1s. 8d.
(27) (*a*) 242, (*b*) 59, " 3$\frac{1}{2}$d.
(28) (*a*) 735, (*b*) 826, " 3s. 4d.
(29) (*a*) 103, (*b*) 161, " 8d.
(30) (*a*) 158, (*b*) 39, " 1s. 0$\frac{1}{2}$d.
(31) (*a*) 38, (*b*) 95, " 7$\frac{1}{2}$d.
(32) (*a*) 39, (*b*) 136, " 19s. 6d.
(33) (*a*) 78, (*b*) 143, at 3s. 9d. per dozen.
(34) (*a*) 840, (*b*) 65, " 1s. 6d. " score.
(35) (*a*) 73, (*b*) 288, " 3s. 3d. " dozen.
(36) (*a*) 180, (*b*) 75, " 4s. 6d. " dozen.

Use the method of nine multiples to find:

(37) The price of 7, 29, 35, 189, 246, 947 tons of coal at 7s. 4$\frac{1}{2}$d. per ton.
(38) The price of 43, 87, 245, 796, 831, 452, 569 yards of cloth at 2s. 7$\frac{1}{2}$d. per yard.
(39) The value in £, s. d. of 49, 123, 765, 849, 1764, 3182 dollars. 1 dollar = 4s. 2d.
(40) The value in £, s. d. of 27, 39, 296, 854, 153, 795 francs. 1 franc = 9$\frac{1}{2}$d.

To find the cost of so many tons, cwts., qrs., lbs., at £1 per ton. £1 per ton = 1s. per cwt. = 3d. per qr. = $\frac{3}{4}$d. for 7 lb. = 1d. for 9 lb. (approximately). Therefore, call the tons £'s, the cwts. shillings, the quarters threepences, and add *either* $\frac{3}{4}$d. for every 7 lb., *or* 1d. for every 9 lb.

For example, 1 ton 1 cwt. 1 qr. 7 lb. at £1 per ton
$= £1,\quad 1s.\quad 3\tfrac{3}{4}d.$
27 tons 18 cwt. 3 qr. 15 lb. at £1 per ton
$= £27,\quad 18s.\quad 10\tfrac{1}{2}d.$

This method may be extended to finding the cost at so many £'s per ton, by finding the cost at £1 per ton, and multiplying by the number of £'s.

For example, 39 tons 13 cwt. 73 lb. at £8, 5s. per ton = £39, 13s. 8d. $\times 8\tfrac{1}{4} = £327, 7s. 9d.$

To find the cost of a lb., having given the cost of a cwt. Suppose the cost of a cwt. $= x$ shillings. Then the cost of a lb. $= \frac{x \times 48}{112}$ farthings $= \frac{x \times 3}{7}$ farthings. We may therefore give the following rule: Express the price per cwt. in shillings, multiply by 3, divide the product by 7, and call the answer farthings.

For example, £3, 7s. 8d. per cwt. $= (67\tfrac{2}{3} \times 3 \div 7)$ farthings per lb. = 29 farthings per lb. $= 7\tfrac{1}{4}$d. per lb.

To find the cost of a lb., having given the cost of a ton. As there are 20 cwt. in a ton, and 20s. in £1, the price per ton in £'s is the same as the price per cwt. in shillings. Hence we may give the following rule: Express the price per ton in £'s, multiply by 3, divide the product by 7, and call the answer farthings.

For example, £25, 13s. 4d. per ton $= (25\tfrac{2}{3} \times 3 \div 7)$ farthings per lb. = 11 farthings per lb. $= 2\tfrac{3}{4}$d. per lb.

Conversely, **To find the price per cwt. or per ton, having given the price of a lb.** Express the price per lb. in farthings, multiply by 7, divide the product by 3, and the answer gives the price per cwt. in shillings, and the price per ton in £'s.

For example, $6\tfrac{3}{4}$d. per lb. $= (27 \times 7 \div 3)$s. per cwt. = 63s. per cwt. = £63 per ton.

Exercise 12.

Find the cost (by the method given) of:

(1) 5 tons 7 cwt. 2 qr. 10 lb. at £6 per ton.
(2) 13 " 16 " 1 " $22\tfrac{1}{2}$ " " £7 "

(3) 18 tons 19 cwt. 3 qr. 26 lb. at £15 per ton.
(4) 7 " 16 " 63 lb. " £3, 5s. "
(5) 12 " 17 " 98 " " £5, 6s 8d. "
(6) 23 " 12 " 48 " " £8, 4s. "

Find the cost of a lb. at

(7) (*a*) 14s., (*b*) 35s., (*c*) 21s., per cwt.
(8) (*a*) 42s., (*b*) 49s., (*c*) £3, 10s., "
(9) (*a*) 10s. 6d., (*b*) 17s. 6d., (*c*) 24s. 6d., "
(10) (*a*) 23s. 4d., (*b*) 32s. 8d., (*c*) 39s. 8d., "
(11) (*a*) 38s. 6d., (*b*) £3, 5s. 4d., (*c*) £4, 2s. 10d., "
(12) (*a*) £28, (*b*) £42, (*c*) £35, per ton.
(13) (*a*) £21, (*b*) £14, (*c*) £49, "
(14) (*a*) £10, 10s., (*b*) £24, 10s., (*c*) £38, 10s., "
(15) (*a*) £30, 6s. 8d., (*b*) £51, 6s. 8d., (*c*) £79, 6s. 8d., "
(16) (*a*) £25, 13s. 4d., (*b*) £88, 13s. 4d., (*c*) £172, 13s. 4d., "

Find the cost of a cwt. at

(17) (*a*) 3d., (*b*) 7½d., (*c*) 1½d., per lb.
(18) (*a*) 4½d., (*b*) 6¾d., (*c*) 8¼d., "
(19) (*a*) 8d., (*b*) 7d., (*c*) 1s. 1d., "
(20) (*a*) 10½d., (*b*) 9¾d., (*c*) 11½d., "
(21) (*a*) 1s. 3½d., (*b*) 1s. 10d., (*c*) 2s. 8d., "

Find the cost of a ton at

(22) (*a*) 6d., (*b*) 9d., (*c*) 10½d., per lb.
(23) (*a*) 2¼d., (*b*) 3¾d., (*c*) 11¼d., "
(24) (*a*) 5d., (*b*) 10d., (*c*) 1s. 4d., "
(25) (*a*) 8½d., (*b*) 11¾d., (*c*) 5¾d., "
(26) (*a*) 1s. 6½d., (*b*) 2s. 1d., (*c*) 2s. 4d., "

PRACTICE.

An **aliquot part** is a quantity which is contained in another quantity an exact number of times. For example, 5s. is an aliquot part of £1, because it is contained in it exactly four times. Similarly, 2 cwt. is an aliquot part of 10 cwt., 9 in. of a yard, and so on.

Strictly speaking, **Practice** is a method of finding, *by means of aliquot parts*, the price, weight, &c. of a number of quantities, having given the price, weight, &c. of one. It is

sometimes convenient, however, to employ fractions which are not aliquot parts, and this we shall occasionally do. The various methods of working questions in Practice will be readily seen from the following examples.

SIMPLE PRACTICE.

(1) *Find the value of* 682 *articles at* £4, 17*s.* 6*d. each.*

	£	s.	d.			£	s.	d.	
	682	0	0	=	value at	1	0	0	each.
			4						
	2728	0	0	=	"	4	0	0	"
10s. $=\frac{1}{2}$ of £1	341	0	0	=	"	0	10	0	"
5s. $=\frac{1}{2}$ of 10s.	170	10	0	=	"	0	5	0	"
2s. 6d. $=\frac{1}{2}$ of 5s.	85	5	0	=	"	0	2	6	"
	3324	15	0	=	"	4	17	6	"

Short Methods.

(2) It is often both convenient and labour-saving to employ subtraction instead of addition in dealing with questions in Practice. In the above exercise, for example, we could have found the value of 682 articles at £5 each, and subtracted from it their value at 2s. 6d. each. This would give us their value at £4, 17s. 6d. each. The working would be shown thus.

	£	s.	d.			£	s.	d.	
	682	0	0	=	value at	1	0	0	each.
			5						
	3410	0	0	=	"	5	0	0	"
2s. 6d. $=\frac{1}{8}$ of £1	85	5	0	=	"	0	2	6	"
	3324	15	0	=	"	4	17	6	"

(3) Instead of taking aliquot parts for 18s., 16s., 14s., and 12s., it will often be more convenient to represent them as $\frac{9}{10}$, $\frac{8}{10}$, $\frac{7}{10}$, and $\frac{6}{10}$ of £1 respectively. All that will then be necessary will be to multiply (mentally) the line which gives the value at £1 by 9, 8, 7, 6, and divide the product by 10 (using the short method of division previously given).

For example, *Find the value of* 639 *articles at* £7, 16*s. each.*

	£			£ s. d.	
	639	=	value at	1 0 0	each.
	7				
	4473	=	"	7 0 0	"
16s. = $\frac{8}{10}$ of £1	·511·2	=	"	0 16 0	"
	4984·2			7 16 0	"
	= £4984 4s.				

(4) *Find the value of* 836$\frac{5}{6}$ *quarters of corn at* £2, 5*s. per quarter.*

Here we can find the value of 836 quarters in the ordinary way, and add the value of $\frac{5}{6}$ of a quarter, or we can proceed as follows:

£$\frac{5}{6}$ = 16s. 8d.; therefore,

	£ s. d.			£ s. d.	
	836 16 8	=	value at	1 0 0	per quarter.
	2				
	1673 13 4	=	"	2 0 0	"
5s. = $\frac{1}{4}$ of £1	209 4 2	=	"	0 5 0	"
	1882 17 6	=	"	2 5 0	"

(5) We might have worked the preceding example by decimals. It will be sufficient if our answer is correct to three decimal places, for any error made by stopping at three decimal places must be less than £$\frac{1}{1000}$; that is, it must be less than a farthing. (Of course, to ensure that *three* places are correct, we must take our working out to *four*.)

$\frac{5}{6}$ = ·8$\dot{3}$; therefore, in preceding example:

	£			£ s. d.	
	836·8333+	=	cost at	1 0 0	per quarter.
	2				
	1673·6667	=	"	2 0 0	"
5s. = $\frac{1}{4}$ of £1	209·2083	=	"	0 5 0	"
	1882·875	=	"	2 5 0	"
	= £1882 17s. 6d.				

(6) We might also decimalise the price per article, and multiply by the number of articles, finding, by the method

of contracted multiplication, an answer true to three decimal places.

For example, *Find the value of* 83691 *articles at* £3, 15*s.* 6¾*d. each.*

N.B.—We must not in this case decimalise £3, 15s. 6¾d. by the approximate method, for the slight error thus made would be multiplied by 83691, and would thus become serious.

£3, 15s. 6¾d. = £3·778125.

£	
83691	
3·7781	25
251073·	
58583·7	
5858·37	
669·528	
8·3691	
1·6738	
·4185	
316195·0594	

= £316195·059 correct to three decimal places.
= £316195, 1s. 2¼d.

(7) Observe the following application of the method of Practice:

Find the dividend on £286, 16*s.* 8*d. at* 12*s.* 6*d. in the* £.

	£	s.	d.			s.	d.	
	286	16	8	= dividend at	20	0	in the £.	
10s. = ½ of £1	143	8	4	=	"	10	0	"
2s. 6d. = ¼ of 10s.	35	17	1	=	"	2	6	"
	179	5	5	=	"	12	6	"

Exercise 13. (*Mental.*)

Find the value at:

(1) 10s. each of 148, 677, 199, 348, 783 articles.
(2) 5s. " 120, 140, 185, 337, 516 "
(3) 4s. " 160, 173, 276, 324, 827 "
(4) 2s. " 850, 392, 504, 696, 791 "

(5)	1s.	each of	600, 720, 552, 456, 684 articles.
(6)	6s. 8d.	"	162, 312, 178, 700, 550 "
(7)	3s. 4d.	"	360, 168, 171, 280, 938 "
(8)	1s. 8d.	"	480, 132, 163, 196, 476 "
(9)	2s. 6d.	"	840, 900, 647, 705, 247 "
(10)	1s. 3d.	"	640, 704, 312, 195, 459 "
(11)	8d.	"	690, 549, 605, 421, 396 "
(12)	6d.	"	960, 484, 723, 851, 195 "

Exercise 14.

Find the values of the following articles:

(1) 173 *at* £1, 10s. ; £2, 5s. ; £3, 4s. ; £5, 6s. 8d. each.
(2) 288 " £3, 3s. 4d. ; £4, 15s. ; £5, 12s. ; £6, 13s. 4d. each.
(3) 324 " £1, 14s. ; £2, 8s. ; £3, 6s. ; £4, 2s. 6d. each.
(4) 189 " £3, 3s. ; £4, 16s. ; £6, 6s. ; £5, 12s. 6d. each.
(5) 403 " £1, 18s. ; 16s. 8d. ; 17s. 6d. ; £2, 19s. each.
(6) 561 " 4s. 6d. ; £2, 1s. ; 9s. ; £2, 7s. 4d. each.
(7) 739 " 11s. 8d. ; £1, 3s. 8d. ; 2s. 8d. ; £5, 14s. 7d. each.
(8) 654 " 13s. 9d. ; £1, 5s. 10d. ; 3s. 8d. ; £2, 2s. 4d. each.
(9) 829 " 15s. 9d. ; £3, 14s. 8d. ; 10s. 4d. ; £1, 9s. 8d. each.
(10) 715 " 16s. 3d. ; £1, 8s. 4d. ; 7s. 2d. ; £4, 14s. 2d. each.
(11) 937 " 12s. 8d. ; £2, 2s. 10d. ; 4s. 10d. ; £1, 4s. 3d. each.
(12) 3657 " £1, 12s. 10½d. ; £2, 7s. 8½d. ; £5, 16s. 3d. each.
(13) 5438 " £3, 11s. 4½d. ; £4, 8s. 6⅜d. ; £2, 19s. 9d. each.
(14) 9245 " £4, 7s. 3¾d. ; £7, 13s. 7½d. ; £9, 18s. 4d. each.
(15) 8592 " £1, 15s. 8½d. ; £3, 3s. 11½d. ; £2, 19s. 11d. each.
(16) 9186 " £3, 14s. 7½d. ; £4, 16s. 9¾d. ; £6, 17s. 10d. each.
(17) 8431 " £9, 9s. 9¾d. ; £12, 19s. 7½d. ; £10, 15s. 8d. each.
(18) 7629 " £11, 11s. 11½d. ; £20, 6s. 5¾d. ; £8, 14s. 10d. each.
(19) 7553 " £7, 16s. 8½d. ; £5, 9s. 6½d. ; £3, 8s. 7d. each.
(20) 2967 " £5, 13s. 9½d. ; £10, 7s. 3½d. ; £6, 9s. 5d. each.

(For examples of a concrete nature see Exercise 16.)

COMPOUND PRACTICE.

Find the value of 7 *cwt.* 3 *qr.* 24 *lb. at* £1, 7*s.* 5*d. per cwt.*

This may be worked in any of the four following ways. The second way is much to be preferred in the working of lengthy exercises. It will be sufficient if the answer is correct to the nearest penny.

FIRST METHOD.

	£	s.	d.			cwt.	qr.	lb.
	1	7	5	=	cost of	1	0	0
			7					
	9	11	11	=	"	7	0	0
2 qr. = ½ of 1 cwt.	0	13	8·5	=	"	0	2	0
1 qr. = ½ of 2 qr.	0	6	10·25	=	"	0	1	0
14 lb. = ½ of 1 qr.	0	3	5·125	=	"	0	0	14
7 lb. = ½ of 14 lb.	0	1	8·5625	=	"	0	0	7
2 lb. = ⅐ of 14 lb.	0	0	5·875	=	"	0	0	2
1 lb. = ½ of 2 lb.	0	0	2·9375	=	"	0	0	1
	10	18	4·25	=	"	7	3	24

SECOND METHOD.

Reduce £1, 7s. 5d. to the decimal of £1, and 7 cwt. 3 qr. 24 lb. to the decimal of 1 cwt.; multiply the first number by the second, correct to 3 decimal places, and reduce to £, s. d.

	£
	1·3708 | 3
	7·9642 | 8
	9·5958
£1, 7s. 5d. = £1·37083	1·2337
7 cwt. 3 qr. 24 lb.	822
= 7 cwt. 3·85714...qr.	55
= 7·96428...cwt.	3
	1
	£10·9176
	= £10·918
	(correct to 3 places)
	= £10, 18s. 4½d.

As alternative methods we might (3) decimalise £1, 7s. 5d. and take aliquot parts of 7 cwt. 3 qr. 24 lb.; or (4) we might decimalise 7 cwt. 3 qr. 24 lb. and take aliquot parts of £1, 7s. 5d. [See (5) in Simple Practice.]

The method of Compound Practice can be applied to the working of such questions as, (*a*) *What quantity of sand can be carried away by* 78 *carts if each cart can carry a load of* 1 *ton* 3 *cwt.* 3 *qr.?* or, (*b*) *Find the produce from* 7 *acres* 2 *roods* 20 *sq. poles at the rate of* 10 *cwt.* 3 *qr.* 16 *lb. per acre.*

The method of working is shown below.

(*a*)

	tons	cwt.	qr.
	78	0	0
2 cwt. = $\frac{1}{10}$ of 1 ton.	7	16	0
1 cwt. = $\frac{1}{4}$ of 2 cwt.	3	18	0
2 qr. = $\frac{1}{2}$ of 1 cwt.	1	19	0
1 qr. = $\frac{1}{2}$ of 2 qr.	0	19	2
	92	12	2

(*b*)

	cwt.	qr.	lb.
	10	3	16
			7
	76	1	0
2 ro. = $\frac{1}{2}$ of 1 ac.	5	1	22
20 sq. po. = $\frac{1}{4}$ of 2 ro.	1	1	$12\frac{1}{2}$
		0	$6\frac{1}{2}$

= 4 tons 3 cwt. $6\frac{1}{2}$ lb.

Exercise 15.

Find the value of:

(1) 4 cwt. 3 qr. 7 lb. *at* 18s. per cwt.
(2) 11 cwt. 16 lb. " £11, 13s. 4d. per ton.
(3) 18 cwt. 24 lb. " £18, 13s. 4d. per ton.
(4) 29 cwt. 2 qr. 15 lb. " £5, 16s. 8d. per cwt.
(5) 38 cwt. 3 qr. 12 lb. " £5, 14s. 4d. per cwt.
(6) 433 tons 7 cwt. 52 lb. " £18, 7s. 6d. per cwt.
(7) 376 tons 3 cwt. 32 lb. " £16, 12s. 6d. per cwt.
(8) 23 cwt. 3 qr. 14 lb. " £2, 11s. 6d. per cwt.
(9) 71 cwt. 2 qr. 14 lb. " £7, 14s. 6d. per cwt.
(10) 30 tons 8 cwt. 1 qr. " £1, 3s. 9d. per ton.
(11) 20 tons 9 cwt. 3 qr. " 18s. 3d. per ton.
(12) 24 tons 3 cwt. 2 qr. 25 lb. " £17, 11s. 8d. per ton.
(13) 16 tons 2 cwt. 3 qr. 11 lb. " £16, 8s. 4d. per ton.
(14) 1 ton 12 cwt. 2 qr. 14 lb. " 17s. 6d. per cwt.
(15) 2 tons 13 cwt. 3 qr. 25 lb. " £2, 16s. per cwt.
(16) 5 tons 11 cwt. 0 qr. 16 lb. " £21 per ton.
(17) 25 yd. 1 ft. 10 in. " 10s. 6d. per yd.
(18) 36 yd. 2 ft. 9 in. " £1, 3s. 4d. per yd.
(19) 59 yd. 2 ft. 7 in. " 15s. 9d. per yd.
(20) 6 m. 25 ch. " £12, 13s. 4d. per mile.
(21) 7 m. 29 ch. 11 yd. " £4 per mile.
(22) 16 m. 37 ch. 11 yd. " £24 per mile.
(23) 157 m. 3 fur. 24 po. " £11, 10s. per mile.
(24) 369 m. 3 fur. 22 yd. " £14, 13s. 4d. per mile.
(25) 2 m. 6 fur. 9 ch. 10 yd. " 18s. 4d. per mile.
(26) 39 ac. 2 ro. 18 po. " £2, 5s. per acre.
(27) 50 ac. 1 ro. 10 po. " £3, 7s. 4d. per acre.
(28) 2 ac. 3 ro. 16 po. " £24, 13s. $4\frac{1}{2}$d. per rood.
(29) 3 ac. 3 ro. 25 po. " £120 per acre.

(30) 17 ac. 2 ro. 25 po. *at* £76, 10s. per acre.
(31) 5 ac. 1 ro. 32 po. " £47, 10s. per acre.
(32) 22 qr. 4 bus. 3 pk. " 5s. 8½d. per bus.
(33) 97 qr. 7 bus. 2 pk. " £2, 5s. 6d. per qr.
(34) 51 bus. 1 pk. 1 gal. 3 qt. " £1, 3s. 9½d. per pk.
(35) 17 cub. yd. 12 cub. ft. 864 cub. in. " £1, 3s. 6d. per cub. yd.
(36) 23 cub. yd. 15 cub. ft. 432 cub. in. " £2, 10s. per cub. yd.

MISCELLANEOUS EXAMPLES IN SIMPLE AND COMPOUND PRACTICE.

Exercise 16.

(1) A bankrupt whose debts amount to £1345 pays 8s. 10½d. in the £. Find the amount of his net assets.
(2) Find the amount realised on £276, 14s. 9d. when a dividend of 13s. 4d. in the £ is received.
(3) A cask of liquid contains 12·875 gallons. What is the value of the liquid at 5s. 6d. the gallon?
(4) Find the cost of 17 cwt. 3 qr. 17½ lb. at £5, 17s. 4d. per ton.
(5) Find the weight of 7328 articles at 3 tons 12 cwt. 1 qr. each.
(6) A farmer has two farms. The first, which contains 275 ac. 1 ro. 10 po., is rented at £3, 11s. 8d. per acre; and the second, which contains 252 ac. 1 ro. 5 po., is rented at £4, 16s. 8d. per acre. What rent should he pay for the two?
(7) What is the sum of the values of 4212⅜ articles at £4, 15s. 7½d. each, and of 493⅝ articles at £2, 15s. 4½d. each?
(8) Find the price of 591 sheep and 57 oxen, the price of a sheep being £2, 16s. 9½d., and of an ox £11, 13s. 6d.
(9) A bankrupt whose liabilities are £3236·875 can pay 13s. 10d. in the £. What are his net assets?
(10) Calculate the duty on 356 cwt. 3 qr. 16 lb. of tea at 37s. 4d. per cwt.
(11) Find a length 156 times 3 yards 2 feet 5 inches (Practice).
(12) What would it cost to make 11 miles 3 furlongs 55 yards of railway at the rate of £3250 a mile?
(13) If the price of a ton of steel be £7, 15s. 6d., what should 132½ tons cost?
(14) Find the value of 3 pockets of hops, each weighing 1 cwt. 3 qr. 14 lb., at £3, 12s. 6d. per cwt.
(15) What will 2 tons 5 cwt. 2 qr. 16 lb. cost at £113, 3s. 4d. a ton?
(16) Find the value of 245 articles at £3, 19s. 9¾d. each; and of 12 ac. 1 ro. 8 po. of land at the rate of £3, 6s. 8d. the acre.

(17) Required the cost of $84\frac{5}{8}$ tons of sugar at 12s. 7d. per cwt.

(18) If 1 acre yields 10 cwt. 3 qr. 20 lb., find, by Practice, the produce of 34 ac. 3 ro. 20 sq. po.

(19) Find, by Practice, 27·61825 times 4 ac. 3 ro. 24 po.

(20) If a mile of cable weighed 107·642 tons, find the weight (in tons) of 12 miles 5 furlongs of the cable.

(21) If a wall costs £215, 8s. 4d. per mile, find the total cost if its length is 17 miles 3 furlongs 118 yards.

(22) What will the wages of 6 labourers amount to for $28\frac{1}{2}$ days at 2s. 3d. each per day?

(23) A person compounded with his creditors by paying them 6s. 3d. in the £. What was lost on an account of £7964?

(24) A bankrupt's debts amount to £1700, and his gross assets to £900, 15s. After paying costs, his creditors receive 5s. 9d. in the £. Find the amount of the costs.

(25) Find the value of 105·85 acres at £32, 1s. 8d. per acre.

(26) The income-tax on £1 is 8d. What is it on an income of £361, 15s., if £160 be allowed to go free of income-tax?

(27) If a person's estate produces a gross income of £1384, 16s. a year, and the rates amount to 2s. $9\frac{1}{2}$d. per £, what is his net annual income?

(28) A man bought 500 lb. of tea at 1s. $11\frac{3}{4}$d. per lb. If 20 lb. be spoiled, what will be the gross gain made by selling the remainder at 2s. 3d. per lb.?

(29) A merchant bought 20 dozen at 4s. $11\frac{1}{2}$d. per bottle; if 3 bottles are spoiled, what will the merchant gain by selling the remainder at 5s. 4d. per bottle?

(30) Find, in the shortest way, the total value of 26 things at £11, 19s. each; 59 things at £5, 12s. 6d. each; and 78 things at £6, 8s. 4d. each.

(31) What is the total value of a cargo of tallow weighing 515 tons at 31s. 3d. per cwt., and of one weighing $753\frac{3}{10}$ tons at £28, 6s. $10\frac{1}{2}$d. per ton?

(32) Find the total cost of 46384 articles at $8\frac{1}{4}$d. each; of $6088\frac{3}{4}$ at £2, 9s. 2d. each; and of 64 tons 12 cwt. 3 qr. 8 lb. at 35s. per cwt.

(33) A contractor agrees to supply a garrison with 1000 cwt. of meat at $5\frac{1}{2}$d. per lb. What will be his gross gain if he buys the meat at £2, 4s. 4d. a cwt.?

(34) What will be the value of 12 miles 3 furlongs 16 poles of telegraph wire at £7 per mile?

(35) Find the total cost of 888 articles at 17s. 6d. per dozen, and of 5 barrels of coffee, each 124 lb., at 140s. per cwt.

(36) What would be the difference between the cost of 7 tons 14 cwt. 2 qr. 26 lb. of sugar at 2⅜d. per lb. and at 2¼d. per lb.?

(37) 9456 articles were bought at 17s. 6d., and sold at £1, 2s. 9d. each. Calculate the profit by Practice.

(38) The gold procured from Australia in 6 months in 1851 amounted to 209096 ounces. In 1861 the New Zealand gold-fields yielded 228292 ounces in the same time. What is the excess in value (at £3, 17s. 10½d. per ounce) of the average monthly return from New Zealand over that from Australia?

(39) A dealer bought 2560 sheep at £2, 4s. 7½d. each, and 500 pigs at 3 guineas a head. He sold 800 of the sheep at £2, 15s. 2d. each, and the remainder of the sheep at 45 shillings each. The pigs he sold at 70 shillings each, and his expenses in the transaction were £200. How much did he gain?

(40) In 1870 the number of men in the Royal Horse Artillery was 1834.

"	"	"	"	Cavalry of the Line	" 7733.
"	"	"	"	Royal Foot Artillery	" 12866.
"	"	"	"	Royal Engineers	" 3879.
"	"	"	"	Foot Guards	" 5220.
"	"	"	"	Infantry of the Line	" 51990.

Find the cost of these corps for that year, the annual cost per man being respectively £55, 6s. 1½d., £52, 11s. 3½d., £32, 6s. 11½d., £31, 5s. 3¾d., £28, 17s. 7¾d., and £26, 3s. 5½d.

THE METRIC SYSTEM.

The metric system of weights and measures has for its basis the *metre*, the unit of length; hence the name. The other units in this system are derived from the metre, and thus all the units of the various measures are connected in a systematic manner.

The metre, or unit of length, was fixed upon (in 1795) because it was supposed to be the ten-millionth part of the distance from the North Pole to the Equator. It has since been discovered that this measurement is not quite correct, but the length of the metre has remained unchanged, and is equal in British measure to 39·3708 inches, or 3 feet 3⅜ inches nearly, or $1\frac{1}{11}$ yards nearly. The metric system is a purely *decimal* system—that is, it proceeds entirely by *tens*.

Multiples of the various units are designated by the Greek prefixes, deca-, hecto-, kilo-, myria-; while **sub-multiples**, or parts, of the various units are designated by the Latin prefixes, deci-, centi-, milli-. Hence,

deca = 10.	**deci** = $\frac{1}{10}$.
hecto = 100.	**centi** = $\frac{1}{100}$.
kilo = 1000.	**milli** = $\frac{1}{1000}$.
myria = 10000.	

Bearing these in mind, it is easy to construct the various tables from the units.

LENGTH.

10 millimetres (mm.)	= 1 centimetre	(cm.)
10 centimetres	= 1 decimetre	(dm.)
10 decimetres	= 1 metre	(m.)
10 metres	= 1 decametre	(Dm.)
10 decametres	= 1 hectometre	(Hm.)
10 hectometres	= 1 kilometre	(Km.)
10 kilometres	= 1 myriametre	(Mm.)

Notes.—(1) The term kilometres is used where we should speak of miles, and metres where we should speak of yards.

(2) 1 kilometre = ·6214 mile. 1 mile = 1·6093 kilometres.
1 metre = 1·0936 yards. 1 yard = ·9144 metre.

(3) For most purposes, it is sufficiently accurate to say that a metre = 3·281 feet or 39·37 inches.

(4) The terms myriametres, hectometres, decametres, and decimetres are rarely used; 3 decimetres would be spoken of as 30 centimetres, 5 decametres as 50 metres, &c.

(5) Centimetres and millimetres are used in scientific calculations.

AREA.

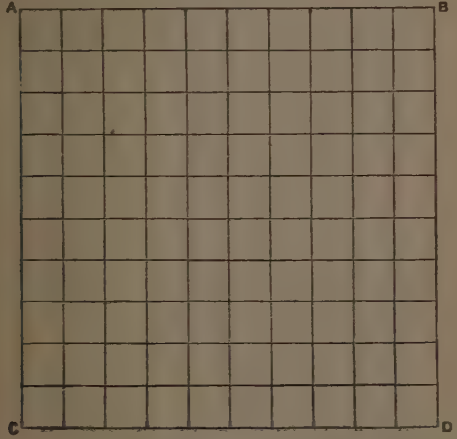
As the *metre* is the unit of length, so the *square metre* (that is, a square the side of which is a metre long) is the unit of area.

Let the figure ACDB on the succeeding page represent a square metre. Then the lines AB and AC will each represent a metre, and if we divide AB and AC into ten equal parts, it is

evident that each of the equal parts will represent a decimetre. If we now complete the figure as shown in the diagram, it is evident that each of the hundred smaller squares thus formed represents a square decimetre. In other words, there are not *ten*, but a *hundred* square decimetres in a square metre.

This principle being borne in mind, the following table will be readily understood.

10^2 or 100	square millimetres (mm.^{2})	= 1	square	centimetre	(cm.^{2})
10^2 „ 100	„ centimetres	= 1	„	decimetre	(dm.^{2})
10^2 „ 100	„ decimetres	= 1	„	metre	(m.^{2})
10^2 „ 100	„ metres	= 1	„	decametre	(Dm.^{2})
10^2 „ 100	„ decametres	= 1	„	hectometre	(Hm.^{2})
10^2 „ 100	„ hectometres	= 1	„	kilometre	(Km.^{2})



Notes.—(1) The terms square decametre and square hectometre are hardly ever used.

(2) For measuring fields, &c., a unit called an *are* (a.) is adopted. The *are* is simply another name for a square decametre or 100 square metres. The only multiple of the *are* in use is the *hectare* (Ha.) (100 *ares*), and the only sub-multiple is the *centiare* (ca.) ($\frac{1}{100}$ of an *are*—that is, a square metre).

(3) Larger areas are expressed in square kilometres.
(4) 1 hectare = 2·471 acres (or $2\frac{1}{2}$ acres, nearly).
1 acre = ·405 hectare.
1 square metre = 1·196 square yards.
1 square yard = ·8361 square metre.

VOLUME.

The unit is the *cubic metre*—that is, a volume of which the length, breadth, and thickness are all exactly a metre. This unit is sometimes called the *stere* (see Notes).

It is evident that a cube which is exactly 10 decimetres (that is, a metre) long, 10 decimetres broad, and 10 decimetres thick can be divided into 1000 smaller cubes, each of which will be a decimetre long, a decimetre broad, and a decimetre thick, or, in other words, each of which will be a cubic decimetre. We thus see that there are 1000 cubic decimetres in a cubic metre.

This principle being borne in mind, the following table will be readily understood.

10^3 or 1000 cubic millimetres (mm.^{3})	= 1	cubic centimetre	(cm.^{3})
10^3 " 1000 " centimetres	= 1	" decimetre	(dm.^{3})
10^3 " 1000 " decimetres	= 1	" metre	(m.^{3})
10^3 " 1000 " metres	= 1	" decametre	(Dm.^{3})
10^3 " 1000 " decametres	= 1	" hectometre	(Hm.^{3})
10^3 " 1000 " hectometres	= 1	" kilometre	(Km.^{3})

Notes.—(1) 1 cubic metre = 1·308 cubic yards (or rather more than $1\frac{3}{10}$ cubic yards).
1 cubic yard = ·7645 cubic metre (or rather more than $\frac{3}{4}$ of a cubic metre).

(2) For measuring firewood, the unit is called the *stere* (s.). The *stere* is simply another name for the cubic metre. The only multiple of the *stere* in use is the *decastere* (Ds.) (10 steres), and the only sub-multiple the *decistere* (ds.) ($\frac{1}{10}$ of a stere).

CAPACITY.

The unit of capacity is the *litre*. A litre is equal to a cubic decimetre, or, in other words, it expresses the capacity of a cubic vessel whose side measures 1 decimetre.

10 millilitres (ml.)	= 1 centilitre	(cl.)
10 centilitres	= 1 decilitre	(dl.)
10 decilitres	= 1 litre	(l.)
10 litres	= 1 decalitre	(Dl.)
10 decalitres	= 1 hectolitre	(Hl.)
10 hectolitres	= 1 kilolitre	(Kl.)

Notes.—(1) The terms millilitres and kilolitres are not used in actual practice.

(2) 1 litre = 1·76 pints (or rather more than 1¾ pints).
1 pint = ·568 litre.

(3) Small quantities of liquids are usually expressed in litres, and large quantities in hectolitres.

WEIGHT.

The unit of weight, called a *gram*, is the weight of a cubic centimetre of pure water at a temperature of 4° C. The weight should be ascertained in a vacuum; otherwise the pressure of the air will affect the result.

10 milligrams (mg.)	= 1 centigram	(cg.)
10 centigrams	= 1 decigram	(dg.)
10 decigrams	= 1 gram	(g.)
10 grams	= 1 decagram	(Dg.)
10 decagrams	= 1 hectogram	(Hg.)
10 hectograms	= 1 kilogram	(Kg.)

Notes.—(1) For heavy weights there are used the
quintal (q.) = 100 kilograms.
tonne or tonneau (t.) = 1000 kilograms.

(2) 1 gram = 15·4323 grains (or 15½ grains, nearly).
1 kilogram (or 'kilo') = 2·2 lb. 1 lb. = ·4536 kilogram.
1 tonne or tonneau = 2204·62 lb. (rather less than the English ton).

(3) The kilogram is the weight most used in commercial transactions. The gram and its sub-multiples are used in scientific calculations.

Reduction, &c.—The metric system being a purely *decimal* one, 'Reduction' is extremely simple.

It is evident, from inspection, that 58 metres equal 5 deca-

metres 8 metres, and that 3678 milligrams equal 3 grams 6 decigrams 7 centigrams 8 milligrams. Similarly, we can readily see that 5 kilograms 9 hectograms 3 grams equal 5903 grams, and that 3 litres 6 decilitres 5 centilitres may be expressed as 365 centilitres, or as 3650 millilitres.

It is also evident that 6 metres 7 decimetres 2 centimetres may be expressed as 6·72 metres, and that 837·09 litres equal 8 hectolitres 3 decalitres 7 litres 9 centilitres.

Similarly, we may say that 8 decagrams 4 grams 7 decigrams 6 milligrams

= 84706 milligrams = 8470·6 centigrams = 847·06 decigrams = 84·706 grams = 8·4706 decagrams = ·84706 hectograms = ·084706 kilogram.

In the metric system, therefore, 'the process of reduction merely consists in the *correct placing of the decimal point.*'

In Addition, Subtraction, &c., it is, as a rule, convenient to express all the quantities in the same denomination before proceeding with the working. For example, to find 87 times 3 litres 8 decilitres 7 centilitres, we can multiply 3·87 litres by 87. The result is 336·69 litres, which may be expressed as 3 hectolitres 3 decalitres 6 litres 6 decilitres 9 centilitres.

Exercise 17. (*Mental.*)

(1) How many decimetres in a metre? How many centimetres?
(2) " " millimetres in a metre? metres in a decametre?
(3) " " metres in a kilometre? in a hectometre?
(4) " " decimetres in a decametre? decametres in a kilometre?
(5) " " decimetres in a hectometre? centimetres in a decametre?
(6) " " centimetres in a hectometre? millimetres in a kilometre?
(7) " " ares in a hectare? How many centiares?
(8) " " square centimetres in a square metre? How many square millimetres?
(9) " " square decimetres in a square decametre? in a square hectometre?

(10) How many square metres in an are? in a hectare?
(11) " " square metres in a square kilometre? How many square hectometres?
(12) " " cubic metres in a cubic decametre? in a cubic hectometre?
(13) " " cubic centimetres in a cubic metre? How many cubic millimetres?
(14) " " cubic decimetres in a cubic metre? in a cubic decametre?
(15) " " decisteres in a stere? in a decastere?
(16) " " cubic decimetres in a stere? decisteres in a cubic metre?
(17) " " litres in a hectolitre? centilitres in a litre?
(18) " " decilitres in a decalitre? in a hectolitre?
(19) " " decilitres in a kilolitre? centilitres in a hectolitre?
(20) " " litres in a cubic metre? How many hectolitres?
(21) " " centilitres in a cubic decimetre? in a cubic metre?
(22) " " grams in a kilogram? centigrams in a gram?
(23) " " decagrams in a hectogram? decigrams in a decagram?
(24) " " centigrams in a hectogram? decigrams in a kilogram?
(25) " " grams in a quintal? in a tonne?
(26) " " decagrams in a quintal? hectograms in a tonne?

EXERCISE 18. (*Mental.*)

Reduce

(1) 13 m. 5 dm. to dm. and then to cm.; 9 dm. 1 cm. to cm. and then to mm.
(2) 7 Km. 8 Hm. 5 m. to m. and dm.; 24 Km. to Hm. and m.
(3) 30·507 Hm. to cm. and mm.; 9 m. 2 dm. 1 mm. to mm. and cm.
(4) 45732 dm. to Hm. and Km.; 90476 mm. to dm. and m.
(5) 4235678 cm. to Km. and Hm.; 654893 dm. to Dm. and Km.
(6) 2193·43 m. to Hm. and dm.; 89·052 Hm. to Km. and m.
(7) 15·3 l. to cl. and ml.; 9·071 Hl. to dl. and l.
(8) 230·4 Dl. to l. and cl.; 80·53 Hl. to Dl. and cl.
(9) 860152 cl. to l. and Dl.; 2563·42 dl. to Dl. and Hl.
(10) 58·325 dl. to l. and Dl.; 7·369 l. to Dl. and Hl.
(11) 29·3 g. to cg. and mg.; 740325 dg. to Hg. and Kg.
(12) 59·07 Kg. to dg. and g.; 89076·4 g. to Hg. and Kg.
(13) 46853 mg. to g. and Dg.; 60532·5 cg. to Dg. and Hg.
(14) 587·46 cg. to g. and Hg.; 1586·25 dg. to g. and Kg.
(15) 73 $m.^2$ to $dm.^2$; 17 $dm.^2$ 28 $cm.^2$ to $mm.^2$.

(16) 856934 cm.^{2} to m.^{2}; 5149326 m.^{2} to Km.^{2}.
(17) 5 m.^{3} 864 dm.^{3} to dm.^{3}; 0·034 dm.^{3} to mm.^{3}.
(18) 82164·3 dm.^{3} to m.^{3}; 1784632 mm.^{3} to dm.^{3}.

ADDITION AND SUBTRACTION.

Exercise 19.

Express the following as metres and find their sum:

(1) 2345 cm., 4217 mm., 692 dm., 175 cm., 4719 mm.
(2) 54 Km., 7 Dm., 43 Hm., 368 m.
(3) 4 Km., 73 dm., 521 Dm., 7432 cm., 781 Hm., 85729 mm.
(4) 761 dm., 842 mm., 9 m., 90 Km., 132 Hm.
(5) 879 cm., 143·5 dm., 40·16 m., 5120 mm., 93·8 dm.
(6) 19·3 Km., 7·2 Hm., 59 m., 3 Km., 5 Dm., 4 dm.

Express the following as litres and find their difference:

(7) 852 Hl. and 35808 l.; 5315 Dl. and 293418 dl.
(8) 88324 l. and 1423 Hl.; 605317 cl. and 73182 l.

Find in grams the sum and the difference of:

(9) 29503 mg. and 842·7 dg.; 84·9 cg. and 703 g.
(10) 6 mg. and 0·12 dg.; 5 Kg. and 3 Hg.
(11) 3·333 g. and 11·7 Hg.; 2 g. and 0·8 Kg.
(12) 4823 mg. and 25 g.; 0·0234 Kg. and 0·25 g.

Find in ares the sum and the difference of:

(13) 18·7 Ha. and 56·3 a.; ·025 Ha. and 152 centiares.

Find in steres the sum and difference of:

(14) 5·48 Ds. and 23·4 s.; 817·5 s. and 92 ds.

Find the value of:

(15) 45 m.^{2} 67 dm.^{2} + 29 m.^{2} 85 dm.^{2} + 16 m.^{2} 9 dm.^{2}.
(16) 120 Km.^{2} 21 Hm.^{2} − 93 Km.^{2} 15 Hm.^{2} 68 Dm.^{2}.
(17) 6 m.^{3} 25 dm.^{3} 108 cm.^{3} 405 mm.^{3} + 370 dm.^{3} 495 mm.^{3} + 860 dm.^{3} 768 cm.^{3} + 12 m.^{3} 42 mm.^{3}.
(18) 25 m.^{3} − 240 dm.^{3} 725 cm.^{3} 309 mm.^{3}.

MULTIPLICATION AND DIVISION.

Exercise 20.

Multiply, giving the answer in metres, litres, or grams as the case may be:

(1) 30·513 m. by 3, 7, 9; 408·457 Km. by 5, 6, 8.
(2) 538·76 dl. by 10, 50, 100; 704·183 Hl. by 12, 84, 144.
(3) 300·502 Dg. by 49, 56, 147; 28·6981 Kg. by 53, 75, 128.

Divide, giving the answer in metres, litres, or grams :

(4) 8·604 m. by 2, 3, 4 ; 490 Km. 16 m. by 6, 7, 9.
(5) 17·5616 Hl. by 8, 49, 112 ; 7786·88 Dl. by 23, 368, 529.
(6) 41063·625 g. by 23, 69, 345 ; 1078 Kg. 50176 cg. by 6, 54, 324.
(7) 58 $m.^2$ 9 $dm.^2$ 37 $cm.^2$ by 8, 24, 120, 480, 500, 1000.
(8) 18 $m.^3$ 65 $dm.^3$ 240 $mm.^3$ by 14, 56, 112, 500, 800, 1000.

CONVERSION FROM BRITISH TO METRIC SYSTEM, AND VICE VERSÂ.

The following are examples of the changing of British weights and measures to their equivalents in the metric system, and *vice versâ*.

(1) How many litres are there in 37 gal. 3 qt. 1 pt. ?

37 gal. 3 qt. 1 pt. = 303 pt.

1 litre = 1·76 pt. ; therefore, in 303 pints there are $\frac{303}{1{\cdot}76}$ litres = 172·16 litres.

(2) How many inches are there in 6·739 metres ?

1 metre = 39·37 in. ; therefore, there are 6·739 × 39·37 in. = 265·3 in.

Short Methods.—The student should keep a careful watch for any opportunities of using short or contracted methods of working, and should only give as many figures in the answer as the data warrant. For example,

(3) How many feet are there in 8·765 metres ?

Since 1 metre = 3·281 ft., there are 8·765 × 3·281 ft., and the answer is reliable to 2 decimal places.

8·765
3·281
26·295
1·753
·701
·009
28·76 feet.

In offices where business is transacted with countries using the metric system, tables are constructed on the 'nine multiple' method to facilitate the calculations. The two tables given

below are most useful. The student will have no difficulty in constructing similar tables for any conversions required.

TABLES FOR CONVERSION OF TONS, CWT., LB. TO KILOGRAMS.

Lb.	Kilos.	Cwt.	Kilos.	Tons.	Kilos.
1	0·45359	1	50·80238	1	1016·0475
2	0·90719	2	101·60475	2	2032·0951
3	1·36078	3	152·40713	3	3048·1426
4	1·81437	4	203·20951	4	4064·1902
5	2·26796	5	254·01188	5	5080·2377
6	2·72156	6	304·81426	6	6096·2852
7	3·17515	7	355·61664	7	7112·3328
8	3·62874	8	406·41902	8	8128·3803
9	4·08233	9	457·22140	9	9144·4279

TABLES FOR CONVERSION OF KILOMETRES TO MILES, AND METRES TO YARDS.

Kilometres.	Miles.	Metres.	Yards.
1	·62138	1	1·09363
2	1·24276	2	2·18727
3	1·86415	3	3·28090
4	2·48553	4	4·37453
5	3·10691	5	5·46817
6	3·72829	6	6·56179
7	4·34968	7	7·65543
8	4·97106	8	8·74907
9	5·59244	9	9·84269

EXAMPLE.—*Express* 7 *tons* 13 *cwt.* 47 *lb. in kilos.; and* 5637 *metres in yards.*

	Kilos.		Yards.
Table gives	7 tons = 7112·3328	Table gives	5000 metres = 5468·17
	10 cwt. = 508·0238		600 " = 656·179
	3 cwt. = 152·4071		30 " = 32·809
	40 lb. = 18·1437		7 " = 7·655
	7 lb. = 3·1751		∴ 5637 " = 6164·81
∴ 7 tons 13 cwt. 47 lb. = 7794·083			

TABLE OF EQUIVALENTS.

LENGTH.

1 inch = 2·539954 cm.	1 cm. = ·3937079 in.
1 foot = 3·047945 dm.	1 dm. = ·3280899 ft.
1 yard = ·914383 m.	1 m. = 1·093633 yd.
1 pole = ·502911 Dm.	1 Dm. = 1·9884237 po.
1 furlong = 2·011644 Hm.	1 Hm. = ·497106 fur.
1 mile = 1·609315 Km.	1 Km. = ·6213824 mile.

AREA.

1 sq. in. = 6·451367 sq. cm.	1 sq. cm. = ·1550059 sq. in.
1 sq. ft. = 9·289968 sq. dm.	1 sq. dm. = ·107643 sq. ft.
1 sq. yd. = ·836097 sq. m.	1 sq. m. = 1·196033 sq. yd.
1 sq. po. = ·252919 ares.	1 are = 119·603326 sq. yd.
1 rood = 1·011678 decares.	1 hectare = 2·471143 acres.
1 acre = ·404671 hectares.	
1 sq. mile = 258·989453 hectares.	

VOLUME.

1 cub. in. = 16·386176 c. cm.	1 cub. cm. = ·061027 cub. in.
1 cub. ft. = 28·315312 c. dm.	1 decistere = 3·531658 cub. ft.
1 cub. yd. = ·7645134 steres.	1 stere = 1·3080215 cub. yd.

WEIGHT.

1 grain = ·6479895 decigram.	1 gram = 15·43234874 grains.
1 lb. troy = ·373242 kilogram.	1 kilo. = 2·204621249 lb. av.
1 lb. avoir. = ·45359265 kilo.	1 quintal = 1·9684118 cwt.
1 stone = 6·350297 kilograms.	1 tonne = ·984206 ton.
1 cwt. = ·5080238 quintal.	
1 ton = 1·0160475 tonnes.	

CAPACITY.

1 pint = ·5679322 litre.	1 litre = 1·7607734 pints.
1 gallon = 4·543458 litres.	= ·2200967 gallon.
1 quarter = 2·907813 hectolitres.	= ·0353166 cub. ft.
1 cub. ft. = 28·315312 litres.	= ·8803867 imp. quart.

Note.—In dealing with lineal, square, and cubic measure in the metric system, the student cannot be too often reminded that a change from one denomination to the **next** denomination is obtained by changing the decimal point

one	place in dealing with	**lineal**	measure,
two	places "	" **square**	"
three	places "	" **cubic**	"

Exercise 21.

(1) How many metres are there in ·025 mile? (1 yard = ·9144 m.)
(2) Express 7 m. 6 dm. in feet. (A metre = 3·281 feet.)
(3) A train ran at the rate of 40 Km. per hour. Express this in miles per hour. (A kilometre = ·6214 mile.)
(4) Express 3700 a. as acres, having given that 1 hectare = 2½ acres.
(5) How many cubic metres are there in 25 cub. yd.? (1 cubic metre = 1·308 cubic yards.) (Answer to three places of decimals.)
(6) How many pints are there in a hectolitre of wine, if a litre = 1·76 pt.?
(7) How many pounds are there in a quintal? (A kilogram = 2·2 lb.)
(8) Express 2½ stones (British) in kilograms. (A kilogram = 2·2 lb.)
(9) Express 12½ gal. as litres, having given that a litre = 1·76 pints.
(10) Express a square mile in hectares. (1 acre = ·405 hectare.)

Exercise 22.

Construct tables of nine multiples for the conversion of:

(1) Yards to metres.
(2) Miles to kilometres.
(3) Acres to hectares.
(4) Hectares to acres.
(5) Steres to cubic yards.
(6) Kilograms to lb.
(7) Gallons to litres.
(8) Litres to pints.
(9) Cwt. to quintals.
(10) Quintals to cwt.

Use the tables to express:

(11) 24 tons 15 cwt. 64 lb. in kilograms.
(12) 2867 kilometres in miles.
(13) 365 yards in metres.
(14) 7498 miles in kilometres.
(15) 653 acres in hectares.
(16) 897 hectares in acres.
(17) 764 steres in cubic yards.
(18) 1267 kilograms in lb.

(19) 6739 kilograms in cwt.
(20) 8975 kilograms in tons.
(21) 4623 gallons in litres.
(22) 1632 litres in pints.
(23) 387 cwt. in quintals.
(24) 563 quintals in cwt.

MISCELLANEOUS EXAMPLES.

Exercise 23.

(1) A tree is 3·04 m. high. Its neighbour is 96 cm. higher. How high is the latter?
(2) Four barrels in a wine-cellar contain respectively 4·05 Hl., 239 l., 34·025 Dl., and 5465 dl. How many hectolitres do they contain altogether?
(3) A dray is loaded with 7 bales weighing respectively 100 Kg., 19800 g., 233 Hg., 3456 Dg., 79·25 Kg., 24·75 Hg., and 3000000 cg. What is the weight of the load in kilograms?
(4) A boiler filled with water weighs 86·25 Kg., and when empty it weighs 15·5 Kg. What weight of water can it contain?
(5) A bath can hold 12 Hl. of water. To fill it 5437 dl. are poured in. How many litres did it already contain?
(6) Glass is made of silica, lime, and potash. In 100 g. of glass there are 61·68 g. of silica and 14·281 g. of lime. Find the number of grams of potash.
(7) A road is 5 Km. long, and 3798 m. of it have been paved. How many Km. are unpaved?
(8) If 125 g. of wool are used in making a pair of socks, how many kilograms would be used in making 80 pairs?
(9) How long will 80 l. of wine last if one drinks 5 dl. a day?
(10) A man walking 8 hours a day travels 1260 Km. in 35 days. How many metres does he walk per hour?
(11) The wheel of a locomotive is 4·5 m. in circumference. How many turns should it make in a second so that the velocity of the locomotive may be 64·8 Km. an hour?
(12) From 4 hectares, 200 square metres are taken. How many hectares (&c.) are left?
(13) A field is 200 m. long and 150 m. broad. How many hectares does it contain?
(14) Express in acres the area of a square the side of which is 50 Hm., having given that a hectare $= 2\frac{1}{2}$ acres.
(15) A square field measures 40·96 Ha. Express the length of a side in metres.

THE COINAGE OF FRANCE.

The monetary unit is the *franc*, a silver piece worth about $9\frac{1}{2}$d.

100 centimes = 1 franc.

Coins used in France.—(1) *Gold-pieces:* 100 francs, 50 fr., 20 fr. (called a louis or a napoleon), 10 fr., 5 fr.

(2) *Silver-pieces:* 5 fr., 2 fr., 1 fr., $\frac{1}{2}$ fr. (or 50 centimes), $\frac{1}{5}$ fr. (or 20 centimes).

(3) *Bronze-pieces:* 10 centimes, 5 c., 2 c., 1 c.

The gold 5-fr. piece, the silver 20-c. piece, and the bronze 2-c. and 1-c. pieces are very little used.

The franc was originally divided into 20 *sous;* hence the 5-c. piece is commonly called a *sou*, and the 10-c. piece is spoken of as 2 *sous*.

The 10-c. piece was formerly called a *decime*—that is, the tenth part of a franc—but the name is now very rarely used.

The monetary system of France being a *decimal* one, the ordinary processes of reduction, &c., are rapidly carried out. For example, we know at a glance that 583 francs 76 centimes = 58376 centimes or 583·76 francs, and that 739851 centimes = 7398·51 francs = 7398 francs 51 centimes.

The following examples of operations in British and French money show clearly the time and labour that would be saved in making calculations if a decimal system were adopted in this country.

Take £98, 17*s.* 8½*d.* *from* £153, 8*s.* 6*d.*

£	s.	d.
153	8	6
98	17	8½
54	10	9½

Take 98 *francs* 75 *centimes from* 153 *francs.*

Francs.
153
98·75
54·25

= 54 fr. 25 c.

Multiply £9, 17*s.* 8½*d.* *by* 53.

£	*s.*	*d.*	
9	17	8½	
		53	
523	18	6½	

26½
424
12)450
37 – 6
371
53
20)93'8
46 – 18
477
523

Multiply 9 *francs* 85 *centimes* *by* 53.

Francs.
9·85 × 53
53
29·55
492·5
522·05
=522 fr. 5 c.

Divide £523, 18*s.* 6½*d.* *by* 53.

£	*s.*	*d.*	
53)523	18	6½(9	
477			
46			=£9, 17s. 8½d.
20			
53)938(17			
53			
408			
371			
37			
12			
53)450(8			
424			
26			
4			
53)106(2			
106			
...			

Divide 522 *francs* 5 *centimes* *by* 53.

Francs.
53)522·05(9·85
477
450 =9 fr. 85 c.
424
265
265
...

Note.—£1 British = 25·2 francs.
1 franc = rather more than 9½d.

Exercise 24.

(1) How many centimes in 10 francs? sous in 5 francs?
(2) How many centimes in a sou? francs in 100 sous?
(3) How many decimes in 100 francs? centimes in 100 sous?
(4) Reduce 84·23 fr. to centimes; 78625 c. to francs.
(5) Reduce 58·50 fr. to sous; 1000 sous to francs.
(6) Take 725 c. from 10 fr.

(7) From 20·25 fr. take 975 c.
(8) Find the product in francs of 35 fr. 45 c. multiplied by 23, 102, 3420; and of 132·07 francs by 73, 211, 1001.
(9) Divide 1574 fr. 64 c. by 4, 12, 48, 50, 96, 100.
(10) If a franc = 9½d., find the value of 371 fr. in British money.
(11) Reckoning 25 fr. to the £, what is the value in British money of 1087 fr. 50 c.?
(12) Express £19, 5s. 1½d. in French money, supposing that a franc = 9½d.

Exercise 25.

(1) How many centimes are there in 85·67 fr.?
(2) Bring 87563 c. to francs.
(3) How many sous are there in 38·64 fr.?
(4) What is the value in francs of 36815 two-sous pieces?
(5) Express £28 sterling in francs. (£1 = 25·2 francs.)
(6) If a franc be taken as worth 9½d., give the price in francs of 100 yards of cloth at 3s. 6¾d. per yard.
(7) A man exchanged 9367 fr. for British money. If the rate of exchange were 25·2 fr. per £, what would he receive?
(8) Towards a debt of 100 fr. the sum of 3250 c. has been paid. How much still remains to be paid?
(9) Find the cost in francs of 12 l. of wine at 60 c. the litre.
(10) If a kilogram of coffee costs 1·75 fr., what will 24 Kg. cost?
(11) A family consumes 3 l. of milk every day at 15 c. the litre. How much should it pay for milk from 1st March to 20th May inclusive?
(12) The wages of a servant are 300 fr. a year. He remains in service only 7 months, and during that time receives 46 fr. 50 c. How much has he still to get?
(13) If the rent of good arable land in Britain is £3, 10s. per acre, find the corresponding rent of similar land in France, expressed in francs per are. (1 hectare = 2·471 acres; £1 = 25·2 francs.)
(14) A merchant bought silk at 3 fr. 75 c. per metre. Find the value of a yard of the silk in British money to the nearest halfpenny. (1 metre = 1·0936 yards; 1 franc = 9½d.)
(15) If the price of tea is half-a-crown a pound, what should I pay in French money for a chest containing a quintal of tea? (1 kilogram = 2·2 lb.; £1 = 25·2 francs.)
(16) A wine merchant bought a cask containing a hectolitre of wine for 750 fr. Find the value in British money of a gallon of the wine. (1 litre = 1·76 pints; 1 franc = 9½d.)

CALCULATION OF EQUIVALENT PRICES.

1. Given price of British unit in £, s. d. to find foreign price per metric unit.

(*a*) Express price as a decimal of £1.

(*b*) Multiply this by the equivalent of the metric unit.

(*c*) Multiply this product by the rate of exchange.

EXAMPLE.—*Find price per metre in francs when* 1 *yard costs* 1*s.* 9½*d. Rate of exchange,* £1 = 25·20 *francs.*

1s. 9½d. = £·0885416. 1 metre = 1·093633 yards.

∴ Price per metre = ·0885416 × 1·093633 × 25·20 francs.

·08854 | 16	·09682 × 25·2 = ·9682 × 2·52
1·09363 | 3	·9682
·08854	2·52
797	1·9364
26	4841
5	194
·09682	2·4399
	= 2·44 francs.

2. Given foreign price per metric unit to find price of British unit in £, s. d.

(*a*) Find the equivalent of the British unit.

(*b*) Multiply this by the foreign price.

(*c*) Divide this product by the rate of exchange.

The answer is expressed as a decimal of £1, and should be found correct to three decimal places.

EXAMPLE.—*Find price per yard when* 1 *metre costs* 2·44 *francs. Rate of exchange,* £1 = 25·20 *francs.*

1 yd. = ·914383 m. ∴ Price per yard = $£\frac{·914383 \times 2·44}{25·2}$.

·91438 | 3	2·2311 ÷ 25·2 = ·22311 ÷ 2·52
2·44	2·52) ·2231 1 (·0885
1·82877	2016
36575	= £·089
3658	225
2·23110	= 1s. 9½d.
	202
	13
	13

Exercise 26.

Use the table of equivalents on page 65.

(1) Find the price per metre in francs and centimes equivalent to 7½d., 1s. 8¼d., 3s. 6½d., and 5s. 6d. per yard, the rate of exchange being £1=25·20 francs.

(2) Find the price per yard in £, s. d. equivalent to 1 franc, 2·75 francs, 3·45 francs, and 6·5 francs per metre, the rate of exchange being £1=25·19 francs.

(3) Find the price per kilogram in francs and centimes equivalent to 3½d., 6d., 1s. 4½d., and 4s. 6d. per lb., the rate of exchange being £1=25·18 francs.

(4) Find the price per lb. in £, s. d. equivalent to ·58 franc, 1 franc, 3·5 francs, and 9·75 francs per kilogram, the rate of exchange being £1=25·18 francs.

(5) Find the price per quintal in francs and centimes equivalent to 9d., 1s. 4½d., 2s. 6d., and 3s. 8¼d. per cwt., the rate of exchange being £1=25·20 francs.

(6) Find the price per cwt. in £, s. d. equivalent to 1·5 francs, 2·75 francs, 3·60 francs, and 6·25 francs per quintal, the rate of exchange being £1=25·20 francs.

(7) Find the price per hectare in francs equivalent to £1, 15s., £2, 7s. 6d., £3, 5s., and £4, 10s. per acre, the rate of exchange being £1=25·18 francs.

(8) Find the price per acre, to the nearest penny, equivalent to 112 francs, 150 francs, 220 francs, and 250 francs per hectare, the rate of exchange being £1=25·20 francs.

PROPOSED BRITISH DECIMAL SYSTEM OF MONEY, WEIGHTS, AND MEASURES.

The adoption of a decimal system of money, weights, and measures has from time to time been discussed by the British Government for nearly a century. Britain is almost the only important country in the world that has not adopted a decimal system. The chief advantages of such a system would be:

(1) Reduction of money, weights, &c. would be enormously simplified. Multiplication and division by awkward numbers

like 28, $5\frac{1}{2}$, $30\frac{1}{4}$, &c. would disappear, and changes from one denomination into another would be carried out by simply moving the decimal point.

(2) Much of the time spent in school in mastering our complicated system would be saved.

(3) Decimals would be universally known.

(4) Foreign traders would understand a decimal system much better, and would be saved endless trouble in converting our money and weights and measures into the equivalents of their own system.

Proposed British Decimal Coinage.—Two systems have been proposed.

(1) The first system is based on the present sovereign as unit, divided as follows:

10 mils (m.) = 1 cent (c.).
10 cents = 1 florin (fl.).
10 florins = 1 pound (£).

The gold coins would remain as at present—the sovereign and half-sovereign; the silver coins would be 20 cents, 10 cents, 5 cents, 2·5 cents, and 1 cent, corresponding to our present double florin, florin, shilling, sixpence, and threepenny-piece; the copper coins would be 4 mils, 2 mils, and 1 mil, corresponding to our penny, halfpenny, and farthing. The values of the new coins would be exactly their present values, except in the cases of the 1 cent, 4 mils, 2 mils, and 1 mil, whose values would be 2·4d., ·96d., ·48d., and ·24d. respectively. The objection which has proved almost, if not quite, fatal to the introduction of this system lies in the difference between these values and the values of the coins they would displace. Suppose the new system were adopted and a coin worth ·96d. replaced our penny; all articles, car tickets, postage-stamps, &c., at present given for a penny would be given for ·96d., entailing a loss to the seller of no less than 4 per cent. The loss on penny postage alone would amount to almost half a million pounds per annum.

(2) The second system is based on the penny as unit. The copper coins would remain as at present; new silver coins would be made worth 2½d., 5d., 10d., 1s. 8d., and 3s. 4d., to take the place of our present threepenny-piece, sixpence, shilling, florin, and double florin; new gold coins would be made worth 8s. 4d. and 16s. 8d., to take the place of our present half-sovereign and sovereign. The great confusion in all money transactions, both home and foreign, that would result, and the enormous expense in changing all tables relating to money from a system where the unit is 20s. to a system where the unit is 16s. 8d., have hitherto proved insuperable difficulties in the introduction of this system.

Exercise 26a.

Express the following in pounds and find their sum:

(1) 283 fl., 1652 c., 9843 m., £14, 3 fl. 4 c. 8 m., £2, 9 c.
(2) 653 m., 987 c., 82 fl., £21, 3 fl. 8 m., £25, 7 fl. 6 c.
(3) £17, 8 c. 7 m., 17 fl., £3, 2 fl. 7 m., 816 c., 9823 m.
(4) 1834 c., 29 fl., £3, 7 fl. 5 c. 2 m., 643 m., £18, 7 fl. 9 m.

Express the following in pounds and find their sum and difference:

(5) 296 c. and 85 fl.; 864 fl. and 7843 m.
(6) £4, 3 fl. 7 c. 4 m. and £1, 7 fl. 8 c. 6 m.; £17, 8 fl. 5 c. and £6, 3 fl. 8 c.
(7) £7, 8 c. 9 m. and £6, 1 fl. 7 m.; £4, 2 fl. 4 m. and £9, 3 c. 7 m.
(8) 185 fl. and £13, 2 fl. 8 c.; 7834 m. and £12, 7 fl. 8 m.

Multiply, giving the answer in pounds, florins, cents, and mils.:

(9) 8743 m. by 3, 7, 9, 12, 84, 147.
(10) £17, 3 fl. 8 c. 5 m. by 3, 5, 8, 13, 216, 849.
(11) £23, 6 c. 9 m. by 2, 6, 7, 21, 582, 756.
(12) £8·753 by 7, 9, 17, 35, 286, 9187.

Divide, giving the answer in pounds, florins, cents, and mils.:

(13) £287, 7 fl. 4 c. 2 m. by 2, 3, 7, 13, 527.
(14) £58, 9 fl. 5 m. by 5, 7, 11, 63, 935.
(15) £8·424 by 3, 8, 18, 648, 702.
(16) £18, 1 c. 8 m. by 6, 7, 9, 693, 1287.

(17) Find the price of 13 articles at £7, 8 fl. 3 c. each.
(18) If £68, 3 fl. 5 c. 3 m. is divided equally among 29 persons, find the share of each.

(19) Find by Practice the value of 3 tons 17 cwt. 77 lb. at £1, 9 fl. 2 c. per ton.

(20) Explain carefully every step in the following:

$$x \text{ pence} = £\frac{x}{240} = \frac{1000\,x}{240} \text{ mils.} = \left(4\,x + \frac{x}{6}\right) \text{mils.}$$

(21) Use the formula in Example 20 to convert 6d., 8d., 9½d., 6¾d., and 7¼d. into mils. (Compare this method with the method given on page 36 for expressing pence and farthings as a decimal of £1.)

(22) Express 4s. 7½d., £1, 3s. 6d., £5, 14s. 7¼d., and £8, 10s. 9¾d. in pounds, florins, cents, and mils.

PROPORTION.

As the principle of Proportion underlies the great majority of calculations, every student should make himself thoroughly conversant with it. It is the most powerful weapon a student can use in attacking questions on Percentages, Brokerage, Commission, Discount, Insurance, Rates and Taxes, Bankruptcy, Profit and Loss, Interest, Mixtures, Partnerships, Exchange, &c.

Ratio.—Ratio is the relation that exists between two quantities of the same kind with regard to magnitude. The ratio of 5 to 6 is written 5 : 6 or $\frac{5}{6}$; 5 and 6 are called the terms of the ratio, 5 being the antecedent and 6 the consequent. A ratio is simply a fraction, and denotes how often one quantity contains another; for example, the ratio of £5 to a guinea is $\frac{£5}{21\text{s.}} = \frac{100}{21}$. Note that no ratio can exist between quantities of different kinds—as, for example, between a ton and a mile. The terms of a ratio must be expressed in the same denomination. In the above example £5 and a guinea are expressed in shillings.

Proportion.—When two ratios are equal the four terms are said to be in proportion. If $\frac{3}{4} = \frac{9}{12}$ or 3 : 4 = 9 : 12, 3, 4, 9, and 12 are said to be in proportion. 3 and 12 are called extremes, and 4 and 9 are called means. Evidently

$3 \times 12 = 4 \times 9$; that is, the product of the extremes = the product of the means. This rule enables us always to find any term in a proportion if we know the other three terms.

If $5 : x = 6 : 7$, then $6x = 5 \times 7$. $\therefore\ 6x = 35$. $\therefore\ x = 5\frac{5}{6}$.

The cost of any commodity is usually proportional to the quantity bought; for example, if 1 lb. of sugar costs 2d., 2 lb. will cost 4d., 3 lb. will cost 6d., and so on.

EXAMPLE.—*If a cwt. of sugar costs 7s., what will 60 lb. cost?*

In this and the following examples we will always take x to represent the unknown term.

As the cost is proportional to the quantity bought, we have

$$\frac{\text{cost of second quantity}}{\text{cost of first quantity}} = \frac{\text{second quantity}}{\text{first quantity}};$$

that is,

$$\frac{x \text{ pence}}{84 \text{ pence}} = \frac{60 \text{ lb.}}{112 \text{ lb.}};$$

$$\therefore x = \tfrac{60}{112} \times 84 \text{ pence}$$

$$= 45\text{d.} = \mathbf{3s.\ 9d.}$$

Fractional Method.—The actual working may be shortened as follows. We first decide what kind of thing is wanted. Here cost is wanted, and we write down the given cost—in this case 84d. We next consider the two quantities and how they will affect the answer, thus: If 84d. is the cost of 112 lb., what will be the cost of 60 lb.?—evidently a smaller sum. Therefore we multiply 84 by $\frac{60}{112}$ and get our answer in pence.

EXAMPLE.—*If I pay* £2, 11*s. for* 34 *lb. of tea, what should I pay for* 196 *lb.?*

Mental Working.—'Cost' wanted, therefore write down 51s. If 51s. is the cost of 34 lb., what is the cost of 196 lb.?—more.

Actual Working.—Cost wanted $= 51 \times \frac{196}{34}$s.

$= 294$s.

$= \mathbf{£14,\ 14s.}$

Note that your answer is always expressed in the same denomination as that of the term first put down.

Unitary Method.—This method, which depends upon finding the price of one article or unit, may be employed,

but it is too cumbrous for business purposes. The previous example would be put down as follows:

34 lb. cost 51s.
$\therefore$ 1 lb. costs $\frac{51}{34}$s.
$\therefore$ 196 lb. cost $\frac{51}{34} \times 196$s. $= 294$s. $=$ **£14, 14s.**

Inverse Proportion.—When quantities are so related that an increase in the one corresponds to a proportional decrease in the other, they are said to be inversely proportional. The same method can be applied.

EXAMPLE.—*If a man takes* 8 *hours to walk a certain distance at the rate of* 3 *miles an hour, how long will he take if his rate is* 4 *miles an hour?*

Mental Working.—Decide what is wanted—time, therefore write down given time—8 hours. If 8 hours is the time when his rate is 3 miles an hour, what is the time when his rate is 4 miles an hour?—less. Therefore multiply by $\frac{3}{4}$.

Actual Working.—$x = 8 \times \frac{3}{4}$ hours.
$= 6$ **hours.**

EXERCISE 27.

(1) If certain books cost £2, 3s. 6d. for 29, find the cost of a gross.
(2) If 27 yd. of cloth cost £5, 14s. 9d., what do 7 yd. cost?
(3) If my income amounts to £77, 8s. 3d. at the end of 5½ months, what is my annual income?
(4) If I sell coal at 7d. per half-cwt., how much should I get for 1 ton 5 cwt.?
(5) What shall I pay for 3 gross of oranges if 9 cost 7½d.?
(6) If pencils are bought at 7s. 6d. the gross, what shall I pay for 4½ doz.?
(7) If the cost of grain is £5, 1s. 6d. for 3 qr. 5 bushels, what is the cost of 4 bushels 2 pecks?
(8) If £3, 17s. 11d. is the cost of 85 lb. of butter, what is the price of 24 lb.?
(9) If £10, 4s. 0½d. is the cost of 83 articles, how many shall I get for £8, 4s. 8½d.?
(10) A man works 313 days a year, and his income is £105, 12s. 9d. How much does he earn per hour if he works 9 hr. per day?
(11) If 54 yd. of cloth cost £5, 19s. 3d., what will 37 yd. cost?
(12) If 11 tons of coal cost £8, 5s., find the price of 7 cwt.

(13) If a cyclist travels 57 miles in 5 hours 4 minutes, how long will he take to travel 33 miles?

(14) If 64 books cost £5, 12s., find the cost of 7 doz. books.

(15) If 26 men do a piece of work in 17 days, how long will 54 men take to do the same work?

(16) If 27 lb. of butter cost £1, 13s. 9d., what will 129 lb. cost?

(17) If a train goes 44 yd. in 3 sec., how far will it go in 3 hr. 20 min. at the same rate?

(18) If 36 men do a piece of work in 35 days, how long will 63 men take to do the same piece of work?

(19) If a man's wages for working 270 days of the year amount to £81, find how much he will earn if he is absent from his work 43 of these days.

(20) If 1 doz. packets of needles cost 10d., and each packet contains 8 needles, how much should I pay for 576 needles?

Compound Proportion.—Sometimes a quantity depends on more than one ratio. In this case we consider each ratio separately, and multiply the given quantity by the correct fraction.

EXAMPLES.—(1) *If* 9 *tons be carried* 25 *miles for* £1, 17*s.* 6*d., how far will* 4 *tons be carried for* £3, 10*s. at the same rate?*

Mental Working.—Distance wanted, ∴ write down the given distance—25 miles.

Next consider the weight. 4 tons will be carried farther than 9 tons at the same rate, ∴ multiply by $\frac{9}{4}$.

Next consider the cost. £3, 10s. will be charged for a longer distance than that for which £1, 17s. 6d. is charged, ∴ multiply by $\frac{3\frac{1}{2}}{1\frac{7}{8}}$.

Actual Working.— $x = 25 \times \frac{9}{4} \times \frac{3\frac{1}{2}}{1\frac{7}{8}} = \frac{\overset{5}{\cancel{25}}}{1} \times \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \times \frac{7}{\underset{1}{\cancel{2}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{15}}} = \mathbf{105}$ **miles.**

(2) *If* 20 *men earn* £85, 10*s. in* 12 *days, working* 9 *hours a day, what should* 15 *men earn in* 16 *days, working* 10 *hours a day at the same rate?*

$$x = £85\tfrac{1}{2} \times \frac{15}{20} \times \frac{16}{12} \times \frac{10}{9} = \frac{171}{2} \times \frac{15}{20} \times \frac{16}{12} \times \frac{10}{9} = \mathbf{£95.}$$

Mental Working.—Money wanted, ∴ write down £85, 10s. 15 men will earn less than 20, ∴ multiply by $\frac{15}{20}$. In 16 days more will be earned than in 12 days, ∴ multiply by $\frac{16}{12}$. In 10 hours more will be earned than in 9 hours, ∴ multiply by $\frac{10}{9}$.

Exercise 28.

(1) If the carriage of 15 cwt. is £6 for 85 miles, what weight should be carried 140 miles for £3, 3s.?

(2) If 16 cwt. be carried 120 miles for £1, 1s., how much will it take to carry 12 cwt. a distance of 210 miles?

(3) If 21 men in 27 days receive £141, 15s. of wages, how many men must work 21 days to earn £157, 10s.?

(4) If 5 furnaces consume 30 tons of coal in 6 days, how many tons should 3 furnaces consume, at the same rate, in 11 days?

(5) How many acres will 16 men reap in $15\frac{1}{2}$ days of 9 hours each, when 12 men reap 16 ac. 3 ro. in 12 days of $8\frac{1}{2}$ hours each?

(6) If a person walking 22 miles a day can perform a journey in 8 days, how long will he take to perform a journey $2\frac{1}{2}$ times as long, walking 19 miles a day?

(7) If 18 men eat 15s. worth of bread in 3 days when wheat is selling at 54s. per quarter, what value of bread will 54 men eat in 27 days when wheat is selling at 50s. per quarter?

(8) If $16\frac{1}{2}$ yd. of velvet 2 ft. 8 in. in width cost £8, 13s. $11\frac{1}{4}$d., how much will be paid for $49\frac{1}{2}$ yd. of the same velvet 28 in. wide?

(9) How much money can be earned by 25 men working 60 days of 8 hr. each, when 4 men working 3 days of 9 hr. each earn £4, 3s. 3d.?

(10) If a man who invests £1127 for $7\frac{1}{2}$ months receives for interest £15, 6s. 8d., how much must he invest for 1 year 2 months so that his interest may amount to £64?

(11) If 2000 books of 60 pages cost £125, how many pages should be in another book, 3500 of which cost £262, 10s.?

(12) If in 19 days of 9 hours each 35 men can build a wall 102 yards long and $9\frac{1}{2}$ ft. high, how high will a wall 320 yards long be which 96 men can build in 70 days of 8 hrs. each?

(13) A piece of work can be done by 90 men in 38 days of $8\frac{1}{2}$ hours each. How many hours per day must 27 men work to do two-thirds of it in 95 days?

(14) If 20 horses and 132 sheep can be fed for 15 days for £88, 2s. 6d., what sum shall be required to feed 14 horses and 241 sheep for 19 days? 2 horses eat as much as 15 sheep.

(15) If a square field can be ploughed in 13 days of $10\frac{1}{2}$ hours each by 3 men, how many days of $9\frac{3}{4}$ hours will it take 8 men to plough another square field whose side is $\frac{1}{3}$rd longer than the first?

(16) If 63 men can dig a trench in 56 days of $9\frac{1}{2}$ hours each, how

many days will 49 men take to dig a trench half as long again, if they work 8 hours per day?

(17) A fort has provisions to last its 1200 men for 83 days, giving each man 2 lb. a day. After 27 days 400 men join them. How much must each get daily if the provisions are to last out the time?

(18) A garrison of 1500 men is supplied with food for 30 days at the rate of 12 oz. per man per day. By what number of men must the garrison be reduced in order that the food may last 45 days, at the rate of 10 oz. per man per day?

(19) A contractor engages 108 men to make a road in 35 days, but he finds that after 8 days only $\frac{1}{7}$th of the work is completed. How many more men must be engaged so that the work may be completed by the time agreed on?

(20) A road contractor engaged to finish $2\frac{1}{2}$ miles of road in 84 days; but after employing 60 men for 54 days, he found that they had finished only 880 yd. How many additional men must he engage so that the work may be finished within the prescribed time?

PERCENTAGES.

The phrase 'per cent.' is a contraction of two Latin words, **per centum**, and means 'for every hundred.' Thus '4 per cent.' means 4 out of every hundred, and can be expressed as $\frac{4}{100}$ or $\frac{1}{25}$.

Instead of 'per cent.,' we often find the letters 'p. c.,' or the sign $\%$.

Percentages are useful for purposes of comparison. If, for example, we wish to compare the death-rates in three villages in which there died 32 out of 1600, 45 out of 1500, and 30 out of 1200 respectively, we can conveniently do so by finding the death-rate **per hundred** in each village—namely, 2, 3, $2\frac{1}{2}$. This at once shows us that the death-rate was lowest in the first village, and highest in the second.

The following examples will show how questions involving percentages are worked. They can all be regarded as exercises in proportion.

(1) *Find* 4 *per cent. of* 650.

This question may be read, '4 are taken out of 100. How many should be taken out of 650?' This is evidently an exercise in proportion, and may be stated thus:

$$x = 4 \times \frac{650}{100} = 26.$$

Short Method.—4 per cent. of anything is $\frac{4}{100}$ or $\frac{1}{25}$ of it; therefore take $\frac{1}{25}$ of $650 = 26$.

In this connection, the student should observe that it is often convenient to change a percentage to its equivalent fractional form. The following equivalents are often used:

Per Cent.	Fraction.	Decimal.	In the £.
1	$\frac{1}{100}$	·01	2·4d.
2	$\frac{1}{50}$	·02	4·8d.
$2\frac{1}{2}$	$\frac{1}{40}$	·025	6d.
4	$\frac{1}{25}$	·04	9·6d.
5	$\frac{1}{20}$	·05	1s. 0d.
10	$\frac{1}{10}$	·1	2s. 0d.
$12\frac{1}{2}$	$\frac{1}{8}$	·125	2s. 6d.
15	$\frac{3}{20}$	·15	3s. 0d.
20	$\frac{1}{5}$	·2	4s. 0d.
25	$\frac{1}{4}$	·25	5s. 0d.
$33\frac{1}{3}$	$\frac{1}{3}$	·33	6s. 8d.
40	$\frac{4}{10}$	·4	8s. 0d.
50	$\frac{1}{2}$	·5	10s. 0d.
$66\frac{2}{3}$	$\frac{2}{3}$	·66	13s. 4d.

(2) *On a certain day* 180 *people travelled by a particular train. On the next day that number was increased by* 5 *per cent. How many travelled by the train on the second day?*

We can find this in two ways.

(1.) We can find 5 per cent. of 180 by either of the methods shown in working question (1), and add it to 180. Or,

(11.) We can read the question thus: '100 became 105; what will 180 become?' As 100 became 105, the required number is evidently $\frac{105}{100}$ of $180 = \frac{21}{20}$ of $180 = \mathbf{189}$.

(3) *During one month* £500 *worth of goods were sold at a certain shop. During the next month* £650 *worth were sold. Find the increase per cent. in the sales.*

Increase on £500 = £150; therefore, the increase on £100 $= \frac{100}{500}$ of £150 $= \frac{1}{5}$ of £150 = £30. **30 per cent.**

(4) *What percentage is* 18 *of* 120?

If 18 are taken out of 120, it is evident that out of 100 we must take $\frac{100}{120}$ of $18 = \frac{5}{6}$ of $18 = 15$. **15 per cent.**

(5) 42 *is* 5 *per cent. of a certain number. Find the number.*

If 42 = 5 per cent., that is, $\frac{5}{100}$ of the number, it is evident that the number must be $42 \times \frac{100}{5} = 42 \times 20 = \mathbf{840}$.

(6) *At the census of* 1861 *the population of a town was* 25000. *At the next census the population was* 24500. *Find the decrease per cent.*

Decrease on 25000 = 500; therefore, the decrease on 100 $= \frac{100}{25000}$ of $500 = \frac{1}{250}$ of $500 = 2$. **2 per cent.**

(7) *A farmer had a flock of* 456 *sheep. His flock was decreased by* $12\frac{1}{2}$ *per cent. How many sheep had he then?*

100 became $87\frac{1}{2}$; therefore, 456 became $\frac{87\frac{1}{2}}{100}$ of 456, $= \frac{175}{200}$ of $456 = \frac{7}{8}$ of $456 = \mathbf{399}$.

(8) *The number of men in a certain regiment at the end of* 1876 *was found to be* 1610, *which was an increase of* 15 *per cent. on the number in it at the end of* 1875. *How many did it contain at the end of* 1875?

The number at the end of 1875 is evidently $\frac{100}{115}$ of the number at the end of 1876 $= \frac{100}{115}$ of $1610 = \frac{20}{23}$ of 1610 $= \mathbf{1400}$.

To convert a fraction into a percentage means to write the fraction with 100 for its denominator. This can always be done mentally by adding a cipher, or if necessary two ciphers,

to both numerator and denominator and then using ordinary division. For example,

$$\frac{33}{40}=\frac{330}{400}=\frac{82\frac{1}{2}}{100}=82\tfrac{1}{2}\ \%.$$

$$\frac{4}{7}=\frac{400}{700}=\frac{57\frac{1}{7}}{100}=57\tfrac{1}{7}\ \%.$$

Sometimes it is easier to multiply both numerator and denominator by a factor other than 10 or 100. For example,

$$\frac{37}{150}=\frac{74}{300}=\frac{24\frac{2}{3}}{100}=24\tfrac{2}{3}\ \%.$$

Exercise 29. (*Mental.*)

Reduce to their equivalent percentages the following fractions:

(1) $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{3}{4}$ $\frac{5}{8}$ $\frac{3}{8}$ $\frac{7}{16}$ $\frac{7}{10}$ $\frac{3}{20}$ $\frac{9}{10}$.

(2) $\frac{8}{25}$ $\frac{9}{20}$ $\frac{11}{50}$ $\frac{13}{50}$ $\frac{17}{20}$ $\frac{9}{40}$ $\frac{11}{20}$ $\frac{1}{25}$ $\frac{11}{40}$ $\frac{24}{25}$.

(3) $\frac{7}{12}$ $\frac{13}{15}$ $\frac{27}{80}$ $\frac{43}{200}$ $\frac{107}{150}$ $\frac{753}{200}$ $\frac{1139}{500}$ $\frac{339}{160}$ $\frac{17}{180}$ $\frac{39}{260}$.

Reduce to their equivalent fractional form the following percentages:

(4) 5% 10% 25% 15% $2\frac{1}{2}$% 4% $7\frac{1}{2}$% $12\frac{1}{2}$%.

(5) $3\frac{1}{3}$% $1\frac{1}{4}$% $6\frac{2}{3}$% $13\frac{1}{3}$% $\frac{1}{2}$% $\frac{3}{4}$% $\frac{1}{8}$% $\frac{5}{16}$%.

(6) 120% 125% $133\frac{1}{3}$% 250% 8·5% 6·25% 1·125% 5·625%.

Write down:

(7) $2\frac{1}{2}$ % of £20; £8, 15s.; £27, 10s.; £500.

(8) 5 " £30; £24, 6s. 8d.; £42, 10s.; £725.

(9) $12\frac{1}{2}$ " £8; £20; £46, 15s.; £367.

(10) 15 " £40; £6, 10s.; £96, 15s.; £450.

(11) 20 " £75; £3, 15s.; £29, 12s. 6d.; £380.

(12) 25 " £24; £8, 16s.; £76, 15s.; £1000.

(13) $33\frac{1}{3}$ " £5; £4, 2s. 6d.; £91, 10s.; £483.

(14) 50 " £13; £2, 1s. 4d.; £85, 16s.; £746.

(15) $66\frac{2}{3}$ " £24; £8, 5s. 9d.; £29; £840.

Exercise 30.

(1) What percentage is 1 in 20; 7 in 84; $1\frac{1}{2}$ in 12?

(2) Find how much per cent. 18 is of 72; 3·5 is of 42; 7·5 is of 5.

(3) What is 5 per cent. of 80; 4 per cent. of 120; $2\frac{1}{2}$ per cent. of 360?

(4) Calculate 10 per cent. on £725; 3½ per cent. on £475; 2¾ per cent. on £213, 6s. 8d.

(5) Of what number is 29, 2 per cent.; and of what number is 57, 150 per cent.?

(6) £86, 12s. is 4 per cent. of a certain sum. What is the sum?

(7) Of what sum is 1500 dollars 3¾ per cent.?

(8) At the rate of 12½ per cent., what amount must be written off for depreciation in the value of machinery worth £9350?

(9) A company made a profit of £740, 5s., and paid a dividend of 5 per cent. Find the amount of the share capital.

(10) On a turnover of £480, 7½ per cent. was gained. Find the amount of profit.

(11) 75 per cent. of the profits of a concern were distributed as dividend. The amount distributed was £270. Find the amount of the profit.

(12) A merchant makes a profit of 8 per cent. on his turnover. If his turnover is £7000 per year, what is his annual income?

(13) The share capital of a company was £15,500. If a profit of £750 were made, and a dividend of 4½ per cent. were paid, how much would be carried forward?

(14) A transaction involving £850 brought in a return of 6 per cent.; another, involving £650, a loss of 4 per cent. Find the loss or gain per cent. on the two transactions together.

(15) A firm's sales during 1898 were £4600, which was an advance of 15 per cent. on those of 1897. Find the sales for 1897.

(16) A firm's expenses are made up as follows: Rent and rates, 13½ per cent.; new machinery, 8 per cent.; wages, 62 per cent.; insurance, ½ per cent.; sundries, 16 per cent. If the amount spent on sundries was £864, find the expenses under each of the other heads.

(17) What sum must A bequeath to B so that B may receive £1000 after a legacy duty of 10 per cent. has been deducted?

(18) What is the freight on 480 bales of cotton weighing 4 cwt. 4 lb. each at ⅜d. per lb. and 5 per cent. additional?

(19) If a person spends 15s. a week and expenses rise 10 per cent., how much ought he to be able to spend in order to spend proportionately as much as before?

(20) An estate with a rental of £8790 is sold for £351600. In order that it may yield the purchaser 2¾ per cent. for his money, how much per cent. must he raise the rent?

(21) The net rental of an estate, after deducting 7d. in the £1 for

income-tax and 5 per cent. on the remainder for the expenses of collecting, is £479, 11s. 10d. What is the gross rental?

(22) The weekly receipts of a railway company are £4800, 15s. 6d. The expenses of working are £13303, 7s. 2d. per calendar month, and their capital is £2000000. What dividend per cent. per annum can they pay?

COMMISSION AND BROKERAGE.

Commission or Brokerage is an allowance made to an agent or broker for buying or selling goods for another, and is generally a certain percentage on the value of the goods bought or sold.

EXAMPLES.—(1) *What is the commission or brokerage on £75 at £1, 15s. per cent.?*

$$\text{Commission} = \frac{1\frac{3}{4}}{100} \text{ of } £75 = £\frac{7}{\cancel{400}\,16} \times \frac{\overset{3}{\cancel{75}}}{1} = £\frac{21}{16} = £1,\ 6s.\ 3d.$$

(2) *An agent sold goods worth £650, and received as commission £29, 5s. What percentage was he allowed?*

$$\text{Allowance} = \frac{29\frac{1}{2}}{650} = \frac{\overset{9}{\cancel{59}}}{\underset{200}{\cancel{1300}}} = \frac{4\frac{1}{2}}{100} = 4\tfrac{1}{2}\ \%.$$

(3) *A broker who is allowed $\frac{1}{8}$ per cent. receives £6, 10s. as brokerage upon a certain transaction. What was the amount of the transaction?*

If he had received £$\frac{1}{8}$, transaction would have been £100. Therefore, as he receives £6, 10s., transaction must be:

$$£100 \times \frac{6\frac{1}{2}}{\frac{1}{8}} = £\frac{100}{1} \times \frac{13}{\cancel{2}} \times \frac{\overset{4}{\cancel{8}}}{1} = £5200.$$

EXERCISE 31.

Find (correct to the nearest farthing) the commission on:

(1) £350 at 5 per cent.
(2) £1125, 10s. at 2½ per cent.
(3) £862, 13s. 4d. at 3 per cent.
(4) £4853, 17s. 6d. at 4½ per cent.
(5) £735, 15s. at 3¼ per cent.
(6) £1568, 7s. 6d. at 2¼ per cent.
(7) I sell goods worth £2369, 5s. 4d. at a commission of 1¾ per cent. What shall I receive?

(8) An agent collected £3650 at a commission of £2, 12s. 6d. per cent. What would he receive for himself?

(9) What would an agent receive for collecting £5148, 10s. at £3, 15s. per cent.?

(10) An agent sold goods worth £6927, 6s. 8d. If his commission was at the rate of £1, 13s. 4d. per cent., what would it amount to?

(11) An agent sold goods worth £357, 10s., and received as commission £17, 17s. 6d. What percentage was he allowed?

(12) Find the percentage allowed to an agent who realises £74, 7s. 6d. on a sale of £2125.

(13) A factor sold goods for his employer to the amount of £333, 6s. 8d., and remitted him £324, 11s. 8d., retaining the balance as commission. How much per cent. did he charge?

(14) An agent, who is allowed 4 per cent. as commission, charges £19 for effecting a sale of goods. What was the value of the goods sold?

(15) A merchant pays his commission agent £2, 10s. per cent. What value of goods must the agent sell in order that he may realise £107, 15s. 3d.?

(16) An agent sold goods for his principal, and made £80, 13s. 4d. of profit. What were the goods sold for, if the agent was allowed $2\frac{1}{2}$ per cent. commission?

Exercise 32.

Find (correct to the nearest penny) the brokerage on:

(1) £475 at $\frac{1}{4}$ per cent.; £1550 at $\frac{1}{8}$ per cent.

(2) £253, 6s. 8d. at $1\frac{1}{2}$ per cent.; £1225 at $\frac{3}{4}$ per cent.

(3) £785 at 12s. 6d. per cent.; £843 at 16s. 8d. per cent.

(4) £1270 at $\frac{5}{8}$ per cent.; £2485 at 17s. 6d. per cent.

(5) A transaction to the amount of £850 is transacted by a broker, who charges $\frac{1}{4}$ per cent. for his trouble. Find the amount of the brokerage.

(6) A broker, who charges $\frac{3}{8}$ per cent., receives 50 guineas for his trouble. What was the amount of the transaction?

(7) A broker sells £3200 of stock, and charges £24. How much is this brokerage per cent.?

(8) A broker, whose charge is $\frac{1}{8}$ per cent., receives 13s. 3d. for buying a certain amount of stock. Find the amount of stock bought.

TRADE DISCOUNT.

It is often the case, when an account is paid promptly, that an allowance is made of so much per cent., or so much in the £. Such an allowance is called Discount. It is often called *ordinary*, *commercial*, or *trade* discount, to distinguish it from *true* discount. It is with the former only that we shall deal here.

EXAMPLES.—(1) *Find the cash price of goods worth* £37, 10s., *discount being allowed at the rate of* 4 *per cent.*

4 per cent. $= \frac{1}{25}$; therefore, discount allowed $= \frac{1}{25}$ of £37, 10s. = £1, 10s.

Therefore, cash price = £37, 10s. − £1, 10s. = **£36.**

Short Method for Mental Work.

20 per cent. = 4s. in the £.		$6\frac{1}{4}$ per cent. = 1s. 3d. in the £.	
15 " = 3s. in the £.		5 " = 1s. in the £.	
$12\frac{1}{2}$ " = 2s. 6d. in the £.		$3\frac{3}{4}$ " = 9d. in the £.	
10 " = 2s. in the £.		$2\frac{1}{2}$ " = 6d. in the £.	
$7\frac{1}{2}$ " = 1s. 6d. in the £.		$1\frac{1}{4}$ " = 3d. in the £.	

Thus the discount on £58, 10s. at $2\frac{1}{2}$ per cent. = $58\frac{1}{2}$ sixpences = £1, 9s. 3d.

(2) *Find the cash price of goods worth* £15, 10s. 3d., *allowing a discount of* $3\frac{3}{4}$ *per cent. on each complete shilling.*

Discount allowed = $15\frac{1}{2}$ ninepences = 11s. $7\frac{1}{2}$d.

Therefore, cash price = £15, 10s. 3d. − 11s. $7\frac{1}{2}$d. = £14, 18s. $7\frac{1}{2}$d.

N.B.—In calculating discount it is often necessary to approximate; for example, see questions (11) *and* (12) *in the following exercise.*

EXERCISE 33.

Find the cash price of goods worth

(1) £22, 10s. at a discount of (*a*) 5 °/₀ (*b*) $3\frac{3}{4}$ °/₀.
(2) £575, 12s. 6d. " " (*a*) $2\frac{1}{2}$ °/₀ (*b*) $7\frac{1}{2}$ °/₀.
(3) £56, 13s. 4d. " " (*a*) $1\frac{1}{4}$ °/₀ (*b*) $6\frac{1}{4}$ °/₀.
(4) £10, 12s. 6d. " " (*a*) 2d. in the shilling, (*b*) $2\frac{1}{2}$ °/₀.
(5) £15, 15s. " " (*a*) 3d. " " (*b*) 5 °/₀.
(6) £57, 15s. " " (*a*) 10 °/₀ (*b*) $7\frac{1}{2}$ °/₀.

(7) £73, 5s.	at a discount of	(*a*) 6d. in the £,	(*b*) 9d. in the £.
(8) £28, 6s. 8d.	" "	(*a*) $3\frac{3}{4}$ °/₀	(*b*) 5 °/₀.
(9) £9, 13s. 4d.	" "	(*a*) $7\frac{1}{2}$ °/₀	(*b*) $2\frac{1}{2}$ °/₀.
(10) £22, 10s.	" "	(*a*) 3d. in the £,	(*b*) 4d. in the £.
(11) £78, 15s. 3d.	" "	$2\frac{1}{2}$ °/₀ (approximately).	
(12) £37, 13s. $2\frac{1}{2}$d.	" "	$12\frac{1}{2}$ °/₀	"

INSURANCE.

Insurance is an agreement on the part of an insurance company, in consideration of a periodical payment, to pay a fixed sum in the event of loss or death. The written agreement is called the **Policy**; the fixed sum the **Amount** of the policy; and the payment, which is usually a percentage of the amount, is called the **Premium.** In life insurance the amount is often payable to the insured on his attaining a certain age, or to his heirs in the event of his death prior to attaining that age. In marine insurance the amount is payable in the event of loss or damage to a ship or its cargo during a certain voyage, or during a certain fixed time. In employers' liability, fidelity guarantee, and casualty insurances special risks are insured, such as accident to servants, loss due to bad debts or burglary, damage to property by lightning, &c.

EXAMPLES.—(1) *Find the yearly premium to insure a property worth* £850 *at* 1*s.* 9*d. per cent.*

1s. 9d. is the premium on £100.

$$\therefore \text{Premium on £850} = 1\text{s. } 9\text{d.} \times \frac{850}{100} = 1\text{s. } 9\text{d.} \times 8\tfrac{1}{2} = 14\text{s. } 10\tfrac{1}{2}\text{d.}$$

(2) *A cargo worth* £1250 *is insured at* 4 °/₀, *the policy duty being* 2s. 6d. °/₀, *and the commission* £$\frac{1}{8}$ °/₀. *Find the amount of the premium.*

Total premium payable to insure £100 = £4 + 2s. 6d. + 2s. 6d.
= £4, 5s.

$$\therefore \text{Required premium} = £4\tfrac{1}{4} \times \frac{1250}{100} = £\frac{17}{4} \times \frac{25}{2} = £\frac{425}{8} = £53,\ 2\text{s. } 6\text{d.}$$

(3) *A man insures his life for* £750, *and pays an annual premium of* £31, 3*s.* 9*d. What rate per cent. is he charged?*

$$\text{Rate per cent.} = £31,\ 3s.\ 9d. \times \frac{100}{750}$$

$$= £31\frac{3}{16} \times \frac{100}{750} = £\frac{499}{16} \times \frac{100}{750} = £4\frac{19}{120} = £4,\ 3s.\ 2d.$$

(4) *Property worth £1018, 10s. is insured at 3 %. Find the amount of the premium so that in the event of the loss of the property both its value and the premium may be recovered.*

Payment of £3 insures a repayment of £100—that is, a *net* repayment of £97.

$$\therefore \text{Required premium} = £3 \times \frac{1018\frac{1}{2}}{97} = £\frac{3}{1} \times \frac{2037}{2} \times \frac{1}{97} = £\frac{63}{2} = £31,\ 10s.$$

Exercise 34.

(1) A man insures his life for £400 at a premium of £3, 6s. 5d. per cent. per annum. Find his annual payment.

(2) A man insures his house, valued at £950, against fire at 1s. 6d. %. Find his annual premium.

(3) A ship worth £2700 is insured for $\frac{3}{4}$ of its value at $2\frac{1}{2}$ %. What premium is paid?

(4) A man insures $\frac{1}{17}$ of a ship, which is valued at £4250, at 3 %, the policy duty being 6s. 8d. %. Find what premium he pays.

(5) A man insures his house, worth £1300, for $\frac{5}{8}$ of its value at 4s. per £100. Find his premium.

(6) A house, worth £1400, is insured for $\frac{3}{4}$ of its value at 5s. per £100. Find the premium.

(7) A man insured his life for £600, and paid an annual premium of £21, 2s. 6d. Find the rate of insurance per cent.

(8) A shipowner insures a vessel worth £1200 for $\frac{2}{3}$ of its value. If he pays £30 as premium, find the rate of insurance.

(9) A man insured his property for £950 at $4\frac{1}{2}$ %. It is partly destroyed, and the insurance company estimate the damage at £396, 15s. If he has paid three annual premiums, what was the loss to the insurance company?

(10) A man who has paid twenty annual premiums on the value of his house at £$\frac{3}{4}$ %, finds he has paid out £105. What is the value of his house?

(11) For what sum must a ship valued at £2895 be insured at $3\frac{1}{2}$ %, so that in the event of its loss both its value and the premium may be recovered?

(12) Find the premium paid to insure goods worth £292, 10s. to cover both their value and the premium at 2½ per cent.

(13) A ship was insured for £800. It was wrecked, but £300 salvage was recovered by the insurance company, whose net loss was £498. Find the insurance rate.

RATES AND TAXES.

Taxes are sums of money raised by the Imperial Government for government purposes, such as upkeep of the army and navy, &c. The chief taxes are :

(1) **Income-Tax** and **Death Duties.** Income-tax is charged at so much per £1 on a man's income. A different rate is charged on income which is earned—that is, received as wages or salary—from that charged on income which is unearned—that is, derived from rents of property or from investments. Certain abatements are allowed on certain incomes. Death duties are charged at so much per cent. on property passing to heirs on account of the death of the owner. The rate varies with the degree of kinship of the heir or heirs, and with the amount of the estate.

(2) **Excise duties** are levied on certain articles manufactured in a country, such as alcoholic liquors, tobacco, &c. They also include licenses to manufacture or sell these. Licenses on motor-cars, dogs, guns, &c., are also excise duties.

(3) **Customs duties** are levied on certain articles imported or exported.

Rates are sums of money raised by the local authorities (County Councils, Parish Councils, Town Councils, Boards of Guardians, &c.) for local purposes, such as the upkeep of roads and streets, poorhouses, asylums, &c. They are usually charged at so much per £1 on the assessed annual value of houses or land. This annual value is called the **rateable value** or **assessment.**

EXAMPLES.—(1) *Find the income-tax payable by a man whose income is* £350 *at* 9*d. per* £1, *an abatement of* £160 *being allowed.*

The man pays tax on £350 − £160 = £190.

$\therefore$ Tax = 9 × 190d. = £7, 2s. 6d.

(2) *Find the rate per £1 which must be imposed to raise £412, 6s. in a parish where the assessment is £6384.*

$$\text{Rate} = £412,\ 6\text{s.} \times \frac{1}{6384} = \frac{98952}{6384}\text{d.} = 1\text{s.}\ 3\tfrac{1}{2}\text{d.}$$

(3) *A man has a net income of £485, 8s. 4d., after paying income-tax at the rate of 10d. in the £ on the excess of his income over £150. Find his gross income.*

On £150 he paid no tax. $\therefore$ £335, 8s. 4d. is the sum to which the taxed part of his income is reduced by subtracting the tax.

£1 of taxed income is reduced to 19s. 2d. by subtracting the tax, what sum is reduced to £335, 8s. 4d. ?

$$x = £1 \times \frac{£335,\ 8\text{s.}\ 4\text{d.}}{19\text{s.}\ 2\text{d.}} = £\frac{335\frac{5}{12}}{\frac{23}{24}} = \frac{\overset{175}{\cancel{4025}}}{\underset{1}{\cancel{12}}} \times \frac{\overset{24}{\cancel{24}}}{\underset{1}{\cancel{23}}} = £350.$$

$\therefore$ His gross income = £350 + £150 = £500.

Exercise 35.

(1) Find the amount raised by a poor-rate of 2s. 6½d. per £1 in a parish valued at £1734, 10s.

(2) What is the school-rate in a parish valued at £3976 if £124, 5s. is collected ?

(3) After paying an income-tax of 10½d. in the £1 a man has £459 left. Find the amount on which he is taxed.

(4) Find the local rates at 11¾d. per £1 on property assessed at £9836.

(5) If the school-rate in a parish is 1s. 5½d. in the pound, what value of property will yield £418, 9s. 4½d. ?

(6) A man whose income is £3700 pays £238, 19s. 2d. as income-tax. Find the rate per £1.

(7) What assessment will produce £221, 5s. 3d. if the rate is 1s. 3½d. in the pound ?

(8) The total rate payable in Edinburgh in 1910 was 5s. 8$\frac{1}{16}$d. in the pound. Find the sum payable by a man who owned a house valued at £36 per annum.

(9) A poor-rate of 7¾d. in the pound raised enough to pay all the poor-law expenses, amounting to £106900, and to carry forward a balance of £10, 17s. 3d. Find the rateable value of the parish.

(10) A man's income is £340, £40 of which is unearned income. He is allowed an abatement of £160. What income-tax does he pay if the rate is 9d. per pound on earned income and 1s. 2d. per pound on unearned income?

(11) Death duty at 4 % is charged on an estate valued at £8450. Find the duty payable.

(12) A man, who is allowed an abatement of £160, finds that his net income is £362, 2s. 6d. Find his gross income if the income-tax rate is 9d. in the pound.

(13) The rateable value of a town is $3\frac{1}{2}$ million pounds. If a rate of 1s. $1\frac{1}{2}$d. be imposed, what sum should be collected? What is the rent of a property on which £125, 14s. 11d. is paid?

(14) In a certain parish the district-rate was 1s. $7\frac{1}{2}$d. in the £, the water-rate 11d. in the £, and the school-rate 1s. $4\frac{1}{2}$d. in the £. Find the total rates payable by a man whose house is assessed at £45.

(15) A householder is charged £6, 11s. 3d. as burgh rates for 3 months, the annual rate being 7s. in the £. Find the amount on which he is rated.

(16) The net rental of an estate, after deducting 7d. in the £ for income-tax and 5 % on the remainder for the expenses of collecting, is £479, 11s. 10d. What is the gross rental?

(17) A man's salary is £400, and he has also an unearned income of £200 per annum. Find his net income if he is allowed an abatement of £160 on his earned income, and the income-tax is 9d. per £ on earned income, and 1s. 2d. per £ on unearned income.

(18) A man pays £8, 17s. 6d. for school-rate, which is charged on $\frac{3}{5}$ of his rental, and is at the rate of 1s. $1\frac{5}{16}$d. in the £. Find the rent of his house.

(19) The rateable value of a parish is £30320. The sum required to be raised by the local rate is £5400. Find the rate to the nearest penny. If this rate is imposed, find the balance to be carried forward.

(20) During a certain year income-tax was 1s. in the £, and during the following year it was 1s. 2d. A company pays a dividend of 7 per cent., less income-tax, on 9500 shares of £2 each for a period including 9 months in the first year and 3 months in the second. What is the total net sum required to pay this dividend?

BANKRUPTCY.

A **Bankrupt** is a man who owes more money than he is able to pay. His **debts** or **liabilities** are the sums of money he is owing to others. He is called the **debtor**; those to whom he owes money are called his **creditors.** His **estate** or **assets** include all moneys possessed by or due to him.

When a man becomes bankrupt or **insolvent,** the law courts usually appoint a judicial factor to settle his affairs. Certain claims on his estate, such as rent, taxes, servants' wages, and the expenses of winding up the estate, are **preferential,** and must be paid in full. What is left is divided among the creditors in proportion to their admitted claims. The debtor is said to pay a **dividend** of so much in the pound.

EXAMPLES.—(1) *A bankrupt's estate is worth* (*net*) £5450 *and his debts are* £6000. *How much can he pay in the pound, and how much will a creditor receive to whom he owes* £1000?

He pays £5450 in place of £6000; ∴ in place of £1 he pays

$$£5450 \times \tfrac{1}{6000} = £\cdot 908\tfrac{3}{3} = 18s.\ 2d.\ \text{per pound.}$$

Creditor receives 18s. 2d. × 1000 = £·908$\frac{3}{3}$ × 1000 = £908, 6s. 8d.

(2) *A bankrupt pays a dividend of* 3*s.* 4*d. in the pound. His net assets are* £1300. *What are his liabilities?*

He pays 3s. 4d. in place of £1; ∴ he pays £1300 in place of

$$£1 \times \frac{£1300}{3s.\ 4d.} = £1 \times \tfrac{1300}{1} \times \tfrac{6}{1} = £7800.$$

(3) *A bankrupt's debts amount to* £6000. *His assets are* £1000, *and* 20 *per cent. of this is spent in winding up the estate. What dividend does he pay?*

£200 is spent in winding up the estate; ∴ £800 is left for the creditors.

∴ Dividend = £800 × $\frac{1}{6000}$ = £$\frac{2}{15}$ = 2s. 8d. per pound.

(4) *A bankrupt's assets are* £467, 10*s.* 8*d.*, *and his liabilities* £1411, 6*s.* 8*d.* *If the preferential claims amount to* £124, 6*s.* 8*d.*, *what dividend can he pay?*

He first pays the preferential claims.

The assets are now = £467, 10s. 8d. − £124, 6s. 8d. = £343, 4s.

The liabilities are now = £1411, 6s. 8d. − £124, 6s. 8d. = £1287.

$$\therefore \text{ Dividend} = £343,\ 4s. \times \tfrac{1}{1287} = \frac{\overset{16}{\cancel{6864}}}{\underset{3}{\cancel{1287}}}s. = 5s.\ 4d.$$

Exercise 36.

(1) A bankrupt's liabilities are £6523, and his net assets £4416, 12s. 3½d. What does he pay in the pound?

(2) A bankrupt's net assets are £664, 1s. 3d., and his liabilities £850. What dividend can he pay, and how much will a creditor receive to whom he owes £125?

(3) A bankrupt pays a dividend of 16s. 8d. in the £. What does a creditor lose to whom he owes £450, 10s. 6d.?

(4) A man pays 15s. 3d. in the £. If his net assets are £701, 10s., what are his liabilities?

(5) A creditor received 16s. 9d. in the £, and thereby lost £390. How much did his debtor owe him?

(6) A bankrupt paid a dividend of 13s. 4d. in the £. Find the loss on a claim for £8700, 12s.

(7) A bankrupt estate paid three successive dividends of 5s., 6s. 8d., and 1s. 8d. in the £. What does a creditor receive in all on debts amounting to £648?

(8) A bankrupt's debts are £836, and his creditors lose £367, 9s. 10d. What dividend does he pay?

(9) A man owes £58 to each of three creditors, and £250 to another. If his assets amount to £265, what dividend can he pay?

(10) A bankrupt's liabilities are £4676, and his assets £3639, 11s. 3d. If the preferential claims amount to £350, what dividend can he pay?

(11) A bankrupt has debts amounting to £3250. A creditor to whom he owes £650 loses £327, 14s. 9½d. What were the bankrupt's assets?

(12) A man's liabilities are £960. He possesses £350 and book-debts amounting to £360, which are valued at 16s. 8d. in the £. What dividend can he pay? (Neglect expenses.)

(13) A bankrupt's liabilities are £3048. He has £2000, 8s. 6d. plus £1024 worth of book-debts, worth 7s. 6d. in the £. What can he pay in the £, neglecting expenses?

(14) A tradesman has the following liabilities: £151; £50, 10s.; £32, 5s.; £145, 5s. In goods and cash he possesses £322,

and he is a creditor for £168, £10, 10s., £4. The first debtor pays him 13s. 4d., the second 14s., and the third 15s. in the £ respectively. Is he bankrupt? If not, what is his credit balance?

(15) A bankrupt's liabilities amount to £900, and his stock is worth £450. He also owns 300 shares worth 13s. 4d. each. What does he pay his creditors in the £, if the expenses of winding up the estate are £33, 6s. 8d.?

(16) A draper fails, and pays a dividend of 15s. 9d. in the £. What does a creditor lose who sold him 870 yd. of cloth at 2s. 6d. per yd.?

(17) A merchant's assets are £2369, 5s., and his debts £3751, 6s. 3d. 5 % of his assets are used in winding up his affairs. How much did he pay in the £?

(18) A merchant sells tea to a tradesman at a profit of 44 per cent.; but the tradesman becomes a bankrupt, and pays only 3s. 4d. in the £. How much per cent. does the merchant lose by the sale?

INVOICES.

Definition.—An **invoice** is a detailed statement sent by a seller of goods to the purchaser at the time the goods are despatched.

In the wholesale trade an invoice should furnish the following particulars:

(*a*) The date.

(*b*) The name, address, and occupation of the seller.

(*c*) Telegraphic address and telephone number of seller.

(*d*) Terms of payment.

(*e*) The number and description, the price per unit, and the total cost of each kind of article despatched.

(*f*) The total cost of all the goods.

(*g*) Any charges made by the seller on account of boxes, crates, hampers, &c.

(*h*) At the foot of the invoice a note should be added stating by what railway or other means of conveyance the goods are being despatched. If they are sent 'carriage paid,' that fact should be stated.

(*i*) Many large firms have printed at the head of their bills an intimation similar to the following: 'No allowance made at settlement for Errors, Returns, or Deductions unless previously admitted by Credit Invoices.' When no such intimation is given, a footnote should be added regarding the return of 'empties.'

(*j*) In the Export and Import trade the number of packages despatched, together with any marks upon the wrappers, is entered upon the extreme left of the invoice.

The following is an example of a 'retail' invoice:

39 Old Street, Birmingham,
Mrs Smythe *July* 12, 1910.
To John Edwards & Co.,
Drapers, &c.

		£	*s.*	*d.*
	8 yards calico @ 6½d. . . .	0	4	4
	2 pairs blankets @ 14s. 6d. .	1	9	0
	3 superfine table-cloths @ 18s. 6½d.	2	15	7½
	1 silk umbrella @ 13s. 11d. .		13	11
	26½ yards braid @ 1½d. . .		3	3¾
		5	6	2¼

When the tradesman receives the money he writes 'Paid' (or 'Received') at the foot of the bill, with his name and the date of payment. If the amount of the bill be £2 or over, the law compels him to put a penny stamp on the paper, and he writes the *receipt*, as it is termed, over that stamp thus:

Paid
John Edwards & Co.
12th July 1910.

The following is a specimen 'wholesale' invoice:

Telegraphic Address: 'Septennial,' Leeds. Telephone 861.

Messrs Thomas & Wright, Aberdeen,
To T. & S. Wightman, Limited,
Cloth Merchants,
3rd July 1910. 536 Every Street, Leeds.

Terms: Three months account net, or 5 % discount for cash.

	£	s.	d.
3 pieces Vicuna cloth, each 30½ yd., @ 6s. 6d. . .	29	14	9
13 pieces Melton cloth, each 42 yd., @ 3s. 9d. . .	102	7	6
80 yd. worsted cloth, @ 6s. 9d.	27	0	0
17 wrappers, @ 1s. 6d. . .	1	5	6
	160	7	9

Carriage paid, per Midland Railway.
Wrappers allowed for when returned.

Exercise 37.

Make out the following invoices in proper form:

(*a*) Retail Invoices.

(1) 10 lb. raisins at 10½d. per lb.; 4½ lb. of almonds at 1s. 4d. per lb.; 15 lb. of grapes at 2s. 6½d. per lb.; 8 lb. of figs at 11d. per lb.; 5⅝ lb. currants at 6d. per lb.; 120 oranges at 1s. 3d. per dozen.

(2) 10½ lb. cheese at 9½d. per lb.; 6 lb. butter at 1s. 9½d. per lb.; 3¼ lb. tea at 2s. 8d. per lb.; 3 score eggs at 1s. 8d. per dozen; 15 lb. ground almonds at 2s. 7½d. per lb.

(3) 3¼ lb. tea at 1s. 8d. per lb.; 1⅜ lb. coffee at 1s. 4d. per lb.; 6¾ lb. loaf-sugar at 5d. per lb.; ¾ lb. butter at 1s. 5d. per lb.

(4) 10 lb. beef at 7¾d. per lb.; 5½ lb. beef at 10½d. per lb.; 6¾ lb. mutton at 9d. per lb.; 14 lb. lamb at 1s. 3d. per lb.; 7⅞ lb. suet at 8d. per lb.; 9 lb. steak at 1s. 1½d. per lb.

(5) 13 lb. beef at 9¾d. per lb.; 7½ lb. mutton at 9½d. per lb.; 17 lb. pork at 8¼d. per lb.; 6½ lb. lard at 10½d. per lb.; 2¼ lb. suet at 8d. per lb.; 9¾ lb. lamb at 1s. 2d. per lb.

(6) 17½ yd. of calico at 10½d. per yd.; 9½ yd. flannel at 1s. 10d. per yd.; 25 reels cotton at 2½d. each; 18 yd. ribbon at 5½d. per yd.; 2½ lb. wool at 2½d. per oz.

(7) 4½ yd. tartan at 6s. 9d. per yd.; 7¼ yd. gray wincey at 2s. 4d. per yd.; 6 pairs socks at 1s. 3d. per pair; 3 pair hose at 4s. 6d. per pair; 1 dozen pairs men's socks at 2s. 9½d. per pair; 5½ yd. flannel at 3s. 11d. per yd. (Deduct 5 per cent. for ready money.)

(8) 8½ yd. silk at 3s. 4d. per yd.; 17 yd. braid at 2½d. per yd.; 19½ yd. merino at 2s. 8½d. per yd.; 6½ yd. velvet at 4s. 9d. per yd.; 18½ dozen buttons at 2½d. a dozen; 6¼ yd. satin at 3s. 6d. per yd.; 7 yd. ribbon at 4⅜d. per yd.; 9¼ yd. lace at 1s. 8d. per yd. (Deduct 1d. in the shilling as discount. Allow discount off the nearest number of shillings.)

(*b*) Wholesale Invoices.

(9) 2½ gross pen-holders at 9d. per dozen; 1750 pencils at 3s. 3d. per 100; 27 dozen rubber bands at 2s. 6d. per gross; 5750 envelopes at 4s. 6d. per 1000; 5025 sheets blotting-paper at 1s. 8d. per 100; 3750 foolscap envelopes at 13s. 10d. per 1000.

(10) 44 reams printing-paper (weight of ream, 60 lb.) at 2½d. per lb.; 6 reams *damaged* ditto (ream weight, 60 lb.) at 2½d. per lb., less 10 per cent.; 29$\frac{4}{20}$ reams glazed blue cover-paper (ream weight, 55 lb.) at 3d. per lb.; 13$\frac{8}{20}$ reams ditto (ream weight, 45 lb.) at 3d. per lb.; 25 reams cartridge-paper (43 lb. to the ream) at 3½d. per lb.; 10 reams ditto (36 lb. to the ream) at 3¼d. per lb.

(11) 50 reams fine printing-paper (ream weight, 66 lb.) at 2⅜d. per lb.; 50 reams ditto (ream weight, 60 lb.) at 2⅜d. per lb.; 24 ditto (ream weight, 67 lb.) at 2¾d. per lb.; 1 ream (60 lb.) damaged ditto at 2¾d. per lb., less 5 per cent.; 50 reams cover-paper (ream weight, 75 lb.) at 2⅝d. per lb.

(12) 13 boxes China tea, each 56 lb., at £8, 9s. 2d. per cwt.; 5 small boxes, each 18 lb. 12 oz., Ceylon tea at 2s. 3d. per lb.; 1 cwt. 84 lb. coffee at £7, 9s. 4d. per cwt.; 8 bags tapioca, each 15 lb., at 3¾d. per lb.; 24 bags flour, each 15½ stones, at 2s. 0½d. per stone.

(13) 17 bars of soap, each $3\frac{1}{2}$ lb., at 3d. per lb. ; 15 boxes of candles, each $14\frac{1}{8}$ lb., at 9d. per lb. ; 4 kegs of soft soap, each 28 lb., at $1\frac{3}{8}$d. per lb. ; 150 qr.-lb. packets washing-powder at $2\frac{1}{2}$d. per lb. ; $15\frac{1}{8}$ gallons paraffin at $1\frac{1}{4}$d. per pint.

(14) 17 pieces canvas, each 85 yd., at 2s. $9\frac{1}{2}$d. per yd. ; 558 fathoms rope at $9\frac{1}{4}$d. per fathom ; 12 spars, each 48 ft. long, at 1s. 9d. per ft. ; 3 anchors, each $19\frac{1}{2}$ cwt., at £5, 6s. 8d. per cwt. ; $5\frac{1}{4}$ dozen iron buckets at 2s. 8d. each. (Deduct $2\frac{1}{2}$ per cent.)

(15) 3 tons 15 cwt. best coal at £1, 2s. 3d. per ton ; 5 tons 8 cwt. kitchen coal at 18s. 9d. per ton ; 9 tons 5 cwt. boiler coal at 13s. 4d. per ton ; 17 sacks coke at 1s. $4\frac{3}{4}$d. per sack ; 500 briquettes at 1s. 9d. per dozen ; 13 sacks firewood at 3s. $2\frac{1}{2}$d. per sack ; 1500 fire-lighters at 5s. 6d. per gross ; $\frac{1}{8}$ ton boncar at 24s. per ton. (Deduct $3\frac{3}{4}$ per cent. for cash.)

(16) 10 salmon, averaging $17\frac{1}{2}$ lb., at 1s. $1\frac{1}{2}$d. per lb. ; 215 codfish, averaging $20\frac{1}{4}$ lb., at 3d. per lb. ; 1560 mackerel at 1s. $6\frac{1}{2}$d. per score ; 2075 herrings at 2s. 6d. per 100 ; 50 score oysters at $10\frac{1}{2}$d. per score. (Deduct $7\frac{1}{2}$ per cent. discount.)

(17) 9 barrels sugar, each $2\frac{1}{2}$ cwt., at $2\frac{1}{2}$d. per lb. ; 14 boxes oranges, each containing 1350, at 5s. 6d. per 100 ; 18 hams, each $11\frac{1}{2}$ lb., at 1s. $1\frac{1}{2}$d. per lb. ; 28 boxes raisins, each 21 lb., at £2, 16s. per cwt. ; 18 dozen eggs at 2 for 3d. ; 9 cheeses, each $18\frac{1}{8}$ lb., at $11\frac{1}{2}$d. per lb. (Deduct $2\frac{1}{2}$ per cent. for cash.)

(18) $5\frac{10}{12}$ gross office ink at 6s. 6d. per gross ; $3\frac{4}{12}$ dozen quarts commercial ink at 5s. 6d. per dozen quarts ; $18\frac{1}{2}$ lb. red sealing-wax at 2s. 3d. per lb. ; $8\frac{1}{4}$ gross bottles blue-black writing ink at 7s. 6d. per gross ; $1\frac{1}{2}$ gross framed slates (6 in. by 4) at $9\frac{1}{4}$d. per dozen ; $8\frac{5}{20}$ reams cream-laid foolscap at 4s. 6d. per ream.

(19) $\frac{1}{2}$ chest of tea, 48 lb. net, at 1s. 6d. per lb. ; 2 cwt. sugar, in 2 bags, at 14s. per cwt. ; 42 lb. coffee, in bag, at 1s. 5d. per lb. ; 6 dozen assorted jams, 2-lb. jars, at 7s. 6d. per dozen ; $\frac{1}{2}$ cwt. tapioca at $1\frac{7}{8}$d. per lb. (Deduct $2\frac{1}{2}$ per cent. discount.)

(20) Invoice for Wool Tops (Combed Wool) : 4 cwt. 3 qr. 13 lb. gross (tare, 9 lb.) ; 5 cwt. 1 qr. 16 lb. gross (tare, 10 lb.) ; 5 cwt. 0 qr. 20 lb. (tare, 10 lb.) ; 4 cwt. 3 qr. 19 lb. (tare, 7 lb.), all at $16\frac{1}{2}$d. per lb. (Deduct 5 per cent. discount.)

(21) To 83 white flooring-boards, each $5\frac{1}{2}$ inches wide (7 of them 23 ft. long ; 3, 22 ft. long ; 7, 21 ft. long ; 19, 20 ft. long ; 14, 19 ft. long ; 1, 18 ft. long ; 9, 17 ft. long ; 11, 16 ft. long ; 4, 15 ft. long ; 5, 14 ft. long ; 3, 13 ft. long), at $15\frac{3}{4}$d. per

sq. yd. ; 18 white deals, each 22 ft. long, at 4½d. per linear ft. ; sawing 1075 ft. at 2s. per 100 ft.

(22) 177 white flooring-boards, each 3 inches wide (16 of them 17 ft. long ; 50, 16 ft. long ; 10, 15 ft. long ; 101, 14 ft. long), at 18d. per sq. yd. ; log of yellow pine, 24 ft. long, each end 18 in. square, at 3s. per cubic ft. ; log of ditto, 36 ft. long, each end 21 in. square, at 3s. 6d. per cubic ft.

(23) 5 tons 3 cwt. 84 lb. gun-metal at £98 per ton ; 87 tons 10 cwt. wrought-iron at £20, 10s. per ton ; 16 tons 7 cwt. tin at £4, 2s. 6d. per cwt. ; 7 cwt. lead at 3½d. per lb.

(24) 18 pieces alpaca, each 86 yd., at 1s. 10½d. per yd. ; 9 pieces, each 103½ yd., at 2s. 4d. per yd. ; 18 dozen yd. silk at 3s. 4½d. per yd. ; 20,000 reels cotton at 9d. per dozen ; 400 umbrellas (200 at 4s. 11d., 50 at 5s. 11d., 150 at 6s. 11d.). (Deduct 2½ per cent. for cash.)

(25) 3½ cwt. sugar at £25 per ton ; 3 qr. 14 lb. tea at £12 per cwt. ; 1¼ cwt. pepper at 1s. 1d. per lb. ; 2 cwt. 3 qr. 21 lb. mustard at 1s. 3d. per lb. ; 2¼ cwt. coffee at 1s. 2½d. per lb. (Allow 7½ per cent. approximately.)

(26) 35 dozen dessert-knives at 22s. per dozen ; 18 carvers at 9s. 6d. each ; 28 gravy-spoons at 7s. each ; 35 dozen dessert-forks at 30s. per dozen ; 24 dozen dessert-spoons at 30s. per dozen ; 17 soup-ladles at 11s. 6d. each. (Deduct 10 % for cash.)

(27) 190 pairs slippers at 2s. 3½d. per pair ; 8½ dozen pairs shoes at 5s. 6d. per pair ; 8½ dozen at 2s. 11d. per pair ; 8 dozen pairs boots at 15s. 6d. per pair ; 10 dozen pairs ladies' boots at 10s. 9d. a pair ; 1½ dozen pairs hunting-boots at 38s. a pair.

(28) 250 pigs at £3, 18s. 9d. each ; 85 turkeys at 8s. each ; 137 geese at 6s. 4d. each ; 94 ducks at 5s. 9d. a couple ; 316 chickens at 3s. a couple ; 1000 eggs at 4½d. a dozen.

PROFIT AND LOSS.

The cost to the manufacturer of making goods, or to the dealer of buying goods, is called the **cost price** or the **prime cost** ; the price at which the manufacturer or the dealer sells the goods he calls the **selling price.** When the selling price is greater than the cost price a **profit** is made ; when the selling price is less than the cost price a **loss** is sustained—the difference between the selling price and the cost price constituting the gain or loss, as the case may be.

N.B.—In each case the profit or loss is considered as profit or loss **on the cost price.**

In order that gains or losses on different kinds or quantities of goods may be compared, it is usual to reduce them to a percentage. That percentage means, not the gain or loss on 100 *articles*, but the gain or loss on £100 *worth* of such articles.

If, for example, a gain of 5 per cent. has been made by selling an article, we know at once that goods which cost £100 must have been sold for £105, or that the cost price, £100, has the corresponding selling price, £105. If a loss of 5 per cent. has been made by selling the article, we know that goods which cost £100 must have been sold for £95, or that the cost price, £100, has the corresponding selling price, £95. Little difficulty will be experienced in working exercises on Profit and Loss by the student who has mastered the chapter on percentages.

Ex. 1. *If I buy goods for* £18 *and sell them for* £22, 10*s.*, *what is my gross gain per cent. ?*

Gain on £18 = £4½; therefore, the gain on £100 equals—

$$£\frac{9}{2} \times \frac{100}{18} = \textbf{25 per cent.}$$

Ex. 2. *A man bought a horse for* £55, *and his expenses in the transaction amounted to* £5. *If he had to sell the horse for* £50, *what did he lose per cent. on his total outlay ?*

His total outlay was £60, on which he lost £10. Therefore, his loss per cent. equals—

$$£10 \times \frac{100}{60} = \textbf{16⅔ per cent.}$$

Exercise 38.

(1) An article which cost 2s. 6d. is sold for 2s. 7½d. What is the gross gain per cent. ?

(2) Required the gross gain per cent. made by selling for £41, 8s. 6d. goods which cost £34, 10s. 5d.

(3) A certain quantity of goods was bought for £29, 14s. 8d., but, having been damaged, they were sold for £26, 0s. 4d. Find the loss per cent.

(4) A draper sells cloth at 14s. a yard, thereby losing 1s. on every yard. What does he lose per cent.?

(5) A contractor agrees to supply meat at 9d. per lb.; he buys the meat at 55s. per cwt., and his expenses are 15s. per cwt. What will be his gain per cent. on his total outlay?

(6) If tea is bought for £9, 16s. per cwt. and sold at 2s. 3d. per lb., what is the gross gain per cent.?

(7) A house built for £664 is sold for £830. What is the gain per cent.? If it had been built for £830 and sold for £664, what would have been the loss per cent.? Why do the rates differ?

(8) A refiner buys sugar at the rate of £12, 10s. a ton. The cost of refining is 10s. 10d. per cwt., and he sells the refined sugar at the rate of 2¾d. a lb. How much does he gain or lose per cent. on his total outlay?

Ex. 3. *What must be the selling price of an article which cost* £20, 17*s.* 6*d. to obtain a gross profit of* 12½ *per cent.?*

If the article had cost £100, it would have to be sold for £112½; therefore, in this case, it must be sold for—

$$£112\tfrac{1}{2} \times \frac{20\frac{7}{8}}{100} = £\frac{225}{2} \times \frac{167}{8} \times \frac{1}{100} = £23,\ 9s.\ 8\tfrac{1}{4}d.$$

Ex. 4. *An article which cost* £2, 10*s. had to be sold at a loss of* 10 *per cent. Find the selling price.*

If the article had cost £100, it would have to be sold for £90; therefore, in this case, it must be sold for—

$$£90 \times \frac{2\frac{1}{2}}{100} = £\frac{90}{1} \times \frac{5}{2} \times \frac{1}{100} = £2,\ 5s.$$

Exercise 39.

(1) The prime cost of goods is £268, 3s. 9d. At what must they be sold to make a gross profit of 20 per cent.?

(2) 5 per cent. was lost by selling an article the prime cost of which was £3, 7s. 6d. At what price was it sold?

(3) If wheat, having been at £2, 10s. 4d. per quarter, rises 6·25 per cent., what is then its price?

(4) If eggs are bought at 5 for 3d., at what price must they be sold per hundred to make a gross gain of 20 per cent.?

(5) Coal is bought at £1, 2s. 6d. per ton. What must be charged per cwt. in order that a gain of 33⅓ per cent. may be made?

(6) A man bought pears at 1s. 2d. a dozen, and sold them so as to gain $28\frac{4}{7}$ per cent. Find the selling price per dozen.
(7) A bookseller buys books at £2, 10s. per dozen, and sells them at a loss of 16 %. What does he charge for each book?
(8) Apples are bought at the rate of 3 for 2d., and sold at a loss of 25 %. What is the selling price of an apple?

Ex. 5. *By selling a house for* £624 *I lose* 4 *per cent. What did the house cost me?*

I sell the house for 96 per cent. of the cost price—that is, 96 per cent. of the cost price = £624; therefore, 100 per cent. of the cost price equals—

$$£624 \times \frac{100}{96} = £650.$$

Ex. 6. *By selling a horse for* £56, 5*s. a man made a gross profit of* $12\frac{1}{2}$ *per cent. Find the cost price.*

$112\frac{1}{2}$ per cent. of the cost price = £56, 5s.; therefore, 100 per cent. of the cost price equals—

$$£56\tfrac{1}{4} \times \frac{100}{112\frac{1}{2}} = £\frac{225}{4} \times \frac{100}{1} \times \frac{2}{225} = £50.$$

Exercise 40.

(1) What is the prime cost of goods which, when sold at £6, 17s. 6d., realised a gross profit of 25 per cent.?
(2) The loss being $12\frac{1}{2}$ per cent. and the selling price £3, 13s. 6d., what was the prime cost?
(3) A man buys land at £1420, and having built houses upon it, sells the land and houses so as to gain 20 per cent. on his whole outlay, and receives £9004, 16s. How much did he spend in building?
(4) What is the cost price of lead per cwt. if the sale of 48 cwt. at £45, 11s. 3d. gives a gross profit of £12, 10s. per cent. on the first outlay?
(5) If 2 tons 3 cwt. 3 qr. be sold for £203, what was the rate of purchase per cwt., the gross profit being 45 per cent.?
(6) If a woman sold eggs at the rate of 35 for 2s. and made a gross profit of 20 per cent., how many did she buy for a shilling?

Ex. 7. *By selling a field for* £280, 10*s. a man lost* $6\frac{1}{2}$ *per cent. What would he have gained or lost per cent. by selling it for* £306?

(1) We might find the cost price by the method employed in

Ex. 5, and then find the gain or loss per cent. by the method employed in Exs. 1 and 2. *Or*—

(2) We may solve the problem by means of one operation. £280, 10s. equals 93½ per cent. of the cost price; therefore, £306 equals—

$$£93\tfrac{1}{2} \times \frac{306}{280\tfrac{1}{2}} = \frac{187}{2} + \frac{306}{1} \times \frac{2}{561} = 102 \text{ per cent.}$$

∴ As cost price was 100 per cent., there is a **gain of 2 per cent.**

Ex. 8. *If by selling for* £1, 2*s.* 2*d. I lose* 2 *per cent., what should I sell for to gain* 5 *per cent.?*

(1) Here again we might find the cost price by the method employed in Ex. 5, and then find the required selling price by the method employed in Ex. 3. *Or*—

(2) We may solve the problem by means of one operation. 98 per cent. of the cost price equals £1, 2s. 2d.; therefore, 105 per cent. of the cost price equals—

$$£1,\ 2s.\ 2d. \times \frac{105}{98} = \mathbf{£1,\ 3s.\ 9d.}$$

Exercise 41.

(1) 5 per cent. was lost by selling at 6s. 4d. What would be gained or lost per cent. by selling at 7s. 4d.?

(2) When the selling price is 4s. 7d. there is a gain of 10 per cent. What should the selling price be in order that a gain of 20 per cent. may be made?

(3) 5 per cent. is lost by selling cloth at 3s. 2d. per yard. At what should it be sold to gain 5 per cent.?

(4) A horse was sold for £79, 10s., and a gain made thereby of 6 per cent. How much would have been lost per cent. if he had been sold for £69?

(5) 33⅓ per cent. is gained by selling at 16s. 8d. What would be gained by selling at 15s. 7½d.?

(6) A bootmaker makes 12½ per cent. gain by selling a pair of boots for 22s. 6d. What should the boots sell for to allow the bootmaker 2½ per cent. gain?

(7) A man sold a field for £138, 12s., and by so doing lost 5½ per cent. on what it cost him. Find what per cent. he would have gained had he sold it for £184, 5s.

(8) A farmer bought two cows at the same price. He immediately sold one for £19, 5s., and thereby gained 15½ per cent.; the other he sold for £20, 10s. What was his profit per cent. in this case?

(9) A man neither gains nor loses by charging 2s. 6d. for a box of figs. What should he charge so as to gain 25 per cent.?

(10) If by selling 8 oranges for 6⅔d. there be a profit of 10 per cent., at what price per dozen must they be sold to gain 21 per cent.?

Ex. 9. *If an article can be manufactured for £25, what must it be marked at to allow a trade discount of 20 per cent. to be given, and yet leave a profit of 5 per cent.?*

As the discount here spoken of is 20 per cent. of the price *marked*, while the profit is 5 per cent. of the *cost* price, it will prevent confusion if the two items are considered separately. We will consider the latter first.

If the cost price be £25, the selling price to gain a profit of 5 per cent. must be £26, 5s. This £26, 5s. is the price at which the article is really sold, after a discount of 20 per cent. is allowed.

To find the price at which the article must be marked so as to permit of this discount of 20 per cent. being made, we proceed thus. £80 would be the *real* (or *net*) *selling price* of goods *marked* £100; therefore, for the *real* (or *net*) *selling price* to be £26, 5s., the goods must be *marked*—

$$£100 \times \frac{26\frac{1}{4}}{80} = £\frac{100}{1} \times \frac{105}{4} \times \frac{1}{80} = \textbf{£32, 16s. 3d.}$$

Ex. 10. *What percentage of the cost price must be put on to cover a discount of 10 per cent. and a profit of 15 per cent.?*

To gain a profit of 15 per cent., goods which cost £100 must be sold for £115 net. When a discount of 10 per cent. is allowed, the net selling price, £90, is obtained for goods marked £100; therefore the net selling price, £115, is obtained for goods marked—

$$£100 \times \frac{115}{90} = 127\frac{7}{9}.$$

$\therefore$ Percentage that must be put on $= \mathbf{27\frac{7}{9}}$ **per cent.**

Exercise 42.

(1) I buy goods for £120. At what price must I mark them so as to have a profit of 10 per cent., after allowing a discount of 10 per cent. on the sale price?

(2) A manufacturer finds that a certain machine costs £75 to produce. What price must he quote so as to be able to allow 10 per cent. discount, and have 20 per cent. profit?

(3) At what price must goods which cost £150 be marked so as

to permit of a discount of 5 per cent., and yet leave 14 per cent. profit?

(4) A tradesman allowed a discount of 5 per cent. off the list price, and still had 10 per cent. profit. How much per cent. was the list price higher than the cost price?

(5) A dining-table cost £6 to make. At what price must it be marked so as to allow 10 per cent. discount and 10 per cent. profit?

(6) What percentage of the cost price must be put on to cover 5 per cent. discount and 12½ per cent. profit?

MISCELLANEOUS EXAMPLES.

Exercise 43.

(1) A quantity of tea is sold for 2s. 1d. per lb., the gain being 10 per cent. and the total gain £6. What is the quantity of the tea?

(2) Bought £13, 6s. 8d. worth of apples at 3s. 4d. per bushel, some of which, being damaged, were entirely lost; for the rest, which were sold at 50 per cent. profit, I received £16. How many bushels were damaged?

(3) A merchant sold goods for £753 more than he gave for them, and cleared 15 per cent. What sum did he buy them for?

(4) A man bought 296 sheep at £1, 7s. 6d. each, and after paying $\frac{8}{11}$ of his outlay for their keep, exchanged them for 37 cows, which he sold at £15 per head. What did he gain per cent. on his total expenditure?

(5) A cwt. of sugar when sold at a gain of 5 per cent. yields 2s. 3½d. more than when sold at a loss of 6 per cent. Find the original cost.

(6) A man sells 110 articles for the cost price of 120. Find his rate per cent. of profit.

(7) A man buys eggs at 1s. 3d. per dozen, and sells them at 11s. 8d. per hundred. Find his gross gain per cent.

(8) A tradesman buys milk at 10d. per gallon, and sells it, diluted with water, at 3d. per quart, thereby making a gross profit of 50 per cent. Find the proportion of water he has added.

(9) A man buys 100 acres of land for £3500, and sells half of it for £1600. At what price per acre must he sell the rest to gain 4½ per cent. on his purchase?

(10) A man buys 4830 yards of cloth at 3s. 4d. per yard. If he sells one-third at a gain of 20 per cent., and the whole at a profit

of £187, 16s. 8d., what is the gain per cent. on the other two-thirds?

(11) A sold goods to B at 8 °/₀ profit. B sold the goods to C for £18, 4s. 6d., making a profit of 12½ °/₀. What did A pay for the goods?

(12) A sold a house to B at a profit of 5 °/₀. B sold the house to C for £780, gaining 14⅔ °/₀ on the transaction. Find what A paid for the house.

(13) Oranges are bought at 7 for 2d., and sold at 8 for 3d. Find the gain per cent., and how many must be sold to gain half a crown.

(14) A grocer in selling goods uses a weight of 15½ oz. instead of a pound weight. What profit per cent. does his dishonesty yield him?

(15) A tea merchant sells tea at 2s. 1d. per lb., and allows a discount of 5 °/₀ for ready cash. Find what the tea cost him per cwt. if he makes a net profit of 18¾ °/₀.

(16) A man sold a horse at a gain of 5 per cent. If the selling price had been £3 more his gain would have been at the rate of 12½ °/₀. What did the horse cost him?

(17) A man sold an article at a profit of 4 °/₀. If he had bought it at 4 °/₀ less and sold it at the same price as before, what would have been his gain per cent.?

(18) An article was sold at a profit of 8 per cent. If it had been bought at 5 per cent. less and sold for 6d. more, the gain would have been 20 per cent. Find its cost price.

(19) A book was sold at a profit of 6 per cent. If it had cost 10 per cent. less and been sold for 3d. less, the gain would have been 16⅔ per cent. Find the cost price of the book.

(20) A bicycle was sold at a loss of 10 per cent. If it had been sold for 22s. 6d. more there would have been a gain of 5 per cent. Find its cost price.

MIXTURES AND ADULTERATIONS.

A merchant often mixes two or more qualities of a commodity. It is necessary to be able (1) to calculate the price of the mixture when he knows the price of each ingredient; (2) to find the proportions in which the different qualities must be mixed to produce a mixture of given value.

EXAMPLES.—(1) *A grocer mixes* 2 *lb. of tea worth* 1*s.* 7*d. per lb.*,

3 lb. worth 2s. per lb., and 5 lb. worth 2s. 4d. per lb. What is the value of a pound of the mixture?

Total value of the tea $= 2 \times 1s.\ 7d. + 3 \times 2s. + 5 \times 2s.\ 4d.$

$= 3s.\ 2d. + 6s. + 11s.\ 8d.$

$= £1,\ 0s.\ 10d.$

Total quantity of tea $= 2 + 3 + 5 = 10$ lb.

$\therefore$ Price of mixture per lb. $= \frac{£1,\ 0s.\ 10d.}{10} = 2s.\ 1d.$

(2) *How must wines worth 28s. and 36s. per gallon be mixed so that the value of the mixture may be 31s. 6d. per gallon?*

The gain on the cheaper quality must equal the loss on the dearer quality.

28s. is 7 sixpences below the selling price.

36s. is 9 " above "

$\therefore$ We must take 9 gallons at 28s. per gallon and 7 gallons at 36s. per gallon. The wines must be mixed in the ratio 9 : 7.

Verification.—9 gallons at 28s. per gallon $= £12,\ 12s.$

7 " 36s. " $= £12,\ 12s.$

$\therefore$ 16 gallons of mixture cost £25, 4s.

$\therefore$ 1 gallon " costs $\frac{£25,\ 4s.}{16} = 31s.\ 6d.$

(3) *How many gallons of water must be mixed with 30 gallons of whisky at 17s. 6d. per gallon, so that a profit of 20 % may be made by selling the mixture at 18s. per gallon?*

18s. corresponds to a gain of 20 %; $\therefore$ the net price of the mixture $= 18 \times \frac{100}{120}s. = 15s.$ per gallon.

Price of whisky (17s. 6d. per gallon) is 5 sixpences above this.

Price of water (nothing) is 30 sixpences below this.

$\therefore$ We mix them in the ratio 30 : 5—that is, 6 : 1.

$\frac{6}{7}$ of the mixture is to be whisky and $\frac{1}{7}$ of it water; $\therefore$ we mix 30 gallons of whisky with 5 gallons of water.

Verification.—30 gallons of whisky at 17s. 6d. per gallon $= £26,\ 5s.$

5 " water costing nothing £0, 0s.

$\therefore$ 35 gallons of mixture cost £26, 5s.

Add 20 % of this sum—that is £5, 5s.

$\therefore$ 35 gallons are sold for £31, 10s.

$\therefore$ 1 gallon is sold for $\frac{£31,\ 10s.}{35} = 18s.$

(4) *In what proportions must tea worth 1s. 7d., 2s., and 2s. 4d. per lb. be mixed to produce a mixture worth 2s. 1d. per lb.?*

The selling price of mixture = 2s. 1d. per lb.

The price of the first quality is 6d. per lb. below this.

" " second " 1d. " "

" " third " 3d. " above this.

The loss on the dearest quality must equal the gain on the cheaper qualities.

∴ We must find multiples of 6, 1, and 3, so that the last product is equal to the sum of the other two. This can generally be done in several ways, and therefore there are usually several alternative answers to a question of this kind. For example,

$$6d. \times 2 + 1d. \times 3 = 3d. \times 5;$$

$$\text{or, } 6d. \times 1 + 1d. \times 3 = 3d. \times 3;$$

$$\text{or, } 6d. \times 3 + 1d. \times 6 = 3d. \times 8,$$

&c.

∴ The following mixtures can be sold at 2s. 1d. per lb.:

2 lb. at 1s. 7d. + 3 lb. at 2s. + 5 lb. at 2s. 4d.

1 " + 3 " + 3 "

3 " + 6 " + 8 "

&c.

Alloys.—In dealing with the fine metals, gold and silver, the oz. troy is the standard weight.

The fineness of gold was formerly estimated in carats, 24 carats representing pure gold; 18-carat gold meant gold containing $\frac{18}{24}$ of its weight of pure gold. The fineness is now estimated in thousandths or millièmes; gold 850 fine means gold containing $\frac{850}{1000}$ of its weight of pure gold.

EXAMPLE.—*A goldsmith has quantities of gold* 18 *carats and* 9 *carats fine respectively. He wishes to make* 10 *oz. of an alloy* 14 *carats fine. How much of each must he take?*

18 carats is 4 above 14 carats.

9 " 5 below 14 "

∴ He must take 5 parts of the former to 4 parts of the latter.

∴ Quantity of 18-carat gold $= \frac{5}{9}$ of 10 oz. $= 5\frac{5}{9}$ oz.

" 9 " $= \frac{4}{9}$ of 10 oz. $= 4\frac{4}{9}$ oz.

EXERCISE 44.

(1) A publican mixes 10 gallons of whisky at 18s. a gallon with 20 gallons at 15s. a gallon. What is a gallon of the mixture worth?

(2) If 2 gallons of wine at 14s. a gallon, 1 gallon at 12s., 2 gallons at 9s., and 4 gallons at 8s. be mixed together, what does a gallon of the mixture cost?

(3) A merchant mixes 6 lb. of tea at 2s. 6d. per lb. with 8 lb. at 2s. 9d. per lb. and 2 lb. at 3s. per lb. What does 1 lb. of the mixture cost him?

(4) A goldsmith mixes 7 ounces of gold 23 carats fine with 13 ounces 19 carats fine. What is the fineness of the compound?

(5) A wine merchant buys two sorts of spirits at the rates of 16s. 6d. and 10s. 6d. per gallon. He mixes them in the proportion of 5 parts of the cheaper to 7 of the dearer. At what rate per gallon must he sell to gain 25 per cent.?

(6) A grocer bought a chest of tea containing 90 lb. at 2s. per lb., another of 40 lb. at 2s. 6d. per lb., and a third of 20 lb. at 4s. per lb. He mixed them together and sold the mixture at 3s. per lb. What was his gross gain per cent.?

(7) A person buys coffee at £5, 12s. 6d. per cwt. and chicory at £2, 5s. 5d. per cwt., and mixes them in the proportion of 2 of chicory to 5 of coffee. He retails the mixture at 1s. 3d. per lb. What is his gross gain per cent.?

(8) A grocer buys teas at 1s. 4d. and 2s. a lb., and mixes them in the proportion of 3:5. Find his gain per cent. by selling the mixture at 2s. 6d. per lb.

(9) In what proportions should a grocer mix teas worth 1s. 3d. and 1s. 11d. per lb. so that he may sell the mixture at 1s. 6d. per lb.?

(10) A grocer mixes coffee worth 1s. 10d. per lb. with chicory worth 7d. per lb. so as to sell the mixture at 1s. 4d. per lb. How are they mixed?

(11) A wine merchant has two qualities of wine worth 35s. and 47s. 6d. per gallon. How should he mix them so as to sell the mixture at 40s. per gallon?

(12) A grocer has teas worth 1s. 3d., 1s. 8d., and 1s. 11d. per lb. How should he mix them so as to sell the mixture at 1s. 6d. per lb.?

(13) A merchant mixes flours worth 27s. 6d., 28s. 3d., 30s. 6d., and 32s. 9d. per quarter so as to sell the mixture at 30s. a quarter. How does he mix them?

(14) A grocer mixes sugars worth $1\frac{1}{2}$d. and $2\frac{1}{2}$d. per lb. so that by selling the mixture at $2\frac{1}{4}$d. per lb. he gains 20 %. In what proportions are they mixed?

(15) How should a wine merchant mix wines worth 28s., 32s., and 37s. 6d. a gallon so as to make a profit of 10 % by selling the mixture at 38s. 6d. per gallon?

(16) A wine merchant buys 84 gallons of whisky at 16s. per gallon; he wishes to gain 25 per cent. on the sale, and yet sell at 15s. per gallon. How many gallons of water must he add to the whisky in order to do this?

(17) Find how much water must be mixed with 40 gallons of spirit, which cost 15s. a gallon, so that by selling the mixture at 12s. a gallon there may be a gain of 10 per cent. on the outlay.

(18) A milkman buys 32 gallons of milk at 7d. per gallon, dilutes it with water, and gains 25 % by selling the mixture at 8d. per gallon. How much water did he add?

(19) A spirit merchant mixes 36 gallons of whisky at 21s. a gallon with 24 gallons at 24s. a gallon. How much water must he add to the mixture so that he may gain $16\frac{2}{3}$ % by selling it at 24s. 8d. per gallon?

(20) A coffee merchant mixed 92 lb. of coffee at £9, 6s. 8d. per cwt. with chicory at £2, 2s. per cwt., and gained $6\frac{1}{4}$ % by selling the mixture at 1s. 5d. per lb. How much chicory did he add?

(21) A woman buys equal quantities of apples at the rate of 2 a penny and 3 a penny respectively, and then mixes them. How much does she gain or lose per cent. by selling them all at 5 for 2d.?

(22) A man buys 20 tons of coal at $4\frac{1}{2}$d. per cwt., 15 tons at 6d. per cwt., and 15 tons at 8d. per cwt., and mixes them. He sells 10 tons of the mixture at 8d. per cwt. What must he sell the remainder for per cwt. to make a net profit of 10 per cent. on his original outlay, allowing 10 per cent. on that outlay for expenses?

(23) A grocer mixed 45 lb. of tea at 1s. 8d. per lb., 95 lb. at 2s. per lb., and 160 lb. at 3s. per lb. At what price per lb. (to the nearest penny) must he sell the mixture so as to make a net profit of 5 per cent. on his original outlay, allowing 15 per cent. on that outlay for expenses of management?

(24) A coal merchant bought 5 tons of coal at 16s. a ton, 10 tons at 18s. a ton, and 5 tons at 20s. a ton; he sold 25 % of the whole at £1, 4s. a ton. What must he charge per cwt. for the remainder so as to gain 25 % on his total outlay?

PROPORTIONAL PARTS AND PARTNERSHIPS.

It is often necessary to divide a sum of money unequally between two persons. For example, divide £100 between A and B, giving A three times as much as B. For every £1 which B gets A gets £3. Therefore, out of every £4 A gets £3 and B £1. That is, A gets $£3 \times \frac{100}{4} = £75$, and B gets $£1 \times \frac{100}{4} = £25$.

EXAMPLE.—*Divide* £350 *among A, B, and C in the ratio* 3 : 4 : 7. Give each one share. A gets £3, B £4, C £7. Total £14.

$$\therefore \text{ A gets } £3 \times \frac{350}{14} = £3 \times 25 = £75,$$

$$\text{B gets } £4 \times \frac{350}{14} = £4 \times 25 = £100,$$

$$\text{C gets } £7 \times \frac{350}{14} = £7 \times 25 = £175.$$

Partnership.—When two or more persons agree to carry on a business together each is called a **Partner,** and the money he invests in the business is his share of the **Capital.** The profits are usually divided between the partners in proportion to their capitals. When a person takes no active part in carrying on the business he is called a sleeping partner, and the others active partners. The active partners get either a fixed sum or a percentage of the profits for their work, and the remaining profit is divided proportionally among the partners.

EXAMPLES.—(1) *A and B enter into partnership, A subscribing* £1100, *and B* £800. *If the profits amount to* £133, *what should each receive?*

$$\text{Total capital} = £1900.$$

$$\therefore \text{ A receives } £133 \times \frac{1100}{1900} = £77.$$

$$\text{B receives } £133 \times \frac{800}{1900} = £56.$$

(2) *A, B, and C enter into partnership, contributing* £625, £1875,

and £2500 respectively. Divide a profit of £1200 among them, giving A 33⅓ % of the profit for managing the business.

A receives £400 as manager; ∴ £800 is left for division.

Total capital = £5000.

$$\therefore \text{ A's share} = £400 + £800 \times \frac{625}{5000} = £400 + £100 = £500.$$

$$\text{B's share} = £800 \times \frac{1875}{5000} = £800 \times \frac{3}{8} = £300.$$

$$\text{C's share} = £800 \times \frac{2500}{5000} = £800 \times \frac{4}{8} = £400.$$

When the element of time enters into the calculation, as when different partners give the use of their money for different periods, we must find the equivalent sum which, invested for equal times, would produce the given profit.

EXAMPLE.—*A, B, and C are partners in a firm, A subscribing £300 for 4 months, B £400 for 5 months, and C £200 for 3 months. Divide a profit of £66, 10s. among them.*

£300 for 4 months is equivalent to £1200 for 1 month.

£400 " 5 " " " £2000 " 1 "

£200 " 3 " " " £600 " 1 "

Total = £3800

$$\therefore \text{ A's share} = £66,\ 10s. \times \frac{1200}{3800} = £\frac{133}{2} \times \frac{12}{38} = £21.$$

$$\text{B's share} = £66,\ 10s. \times \frac{2000}{3800} = £\frac{133}{2} \times \frac{20}{38} = £35.$$

$$\text{C's share} = £66,\ 10s. \times \frac{600}{3800} = £\frac{133}{2} \times \frac{6}{38} = £10,\ 10s.$$

EXERCISE 45.

(1) Gunpowder consists of nitre, carbon, and sulphur in the ratio 15 : 3 : 2. What weight of each of these substances is contained in 1 cwt. of gunpowder?

(2) A penny is made of an alloy of copper, tin, and zinc in the ratio of 95 : 4 : 1. Find the weight of each of these in £10 worth of coppers, if 3 pennies weigh 1 oz.

(3) A and B are partners, A contributing £1200, and B £750. Divide a profit of £136, 10s. between them in proportion to their capitals.

(4) A, B, and C contribute £850, £720, £540 respectively to

the capital of a business. What should each receive out of a profit of £221, 11s.?

(5) A, B, and C enter into partnership with £3000, £2600, and £1900 respectively, agreeing to divide the profit in the ratio of their capitals. A receives £397, 10s. Find the total profit, also B's and C's shares.

(6) A and B enter into partnership, A contributing £750, and B £360. B acts as manager, for which he receives a salary of £100. How should a gross profit of £194, 7s. be divided between them?

(7) A, B, and C are copartners, investing £4000, £5000, and £1500 respectively in the business. The gross profit is £2625, of which C receives 18 % as manager, and the balance is divided in proportion to their capitals. Find the share of each.

(8) A starts business with £850. At the end of 4 months he is joined by B, who invests £1000. At the end of the year a profit of £196 has been made. Find the share of each to the nearest penny.

(9) A and B rent a field for a month for £5. A puts in 120 sheep for 4 weeks, and B puts in 80 sheep for 2 weeks. What should each pay?

(10) In a copartnership A supplies £240 for 12 months, B supplies £360 for 10 months, C £400 for 8 months, and D £720 for 6 months. Find what each should receive out of a profit of £297, 10s.

(11) A and B invest £2300 and £2700 respectively in a business. The gross profit amounts to £900, of which 5 % is carried to a reserve account and £50 paid for rent and taxes. What is each partner's share of the remaining profit?

(12) Apportion a feu-duty of £16, 16s. among 3 houses valued at £580, £620, £480, in the ratio of their values.

(13) A, B, and C contributed £1300, £800, and £1500 to buy a ship. The ship is insured for £3000 at 5 %. How much of the premium should each pay?

(14) A, B, C, and D purchase a ship. A pays for 6 shares, B for 5, C for 3, and D for 4. They gain £364, 17s. 6d. How much should each receive?

(15) Divide £57, 15s. $3\frac{1}{2}$d. among A, B, and C so that A may get twice as much as B, and B twice as much as C.

(16) Divide £28, 4s. among A, B, and C so that for every shilling given to A, B may have 2 shillings and C 3 shillings.

(17) Prize money to the value of £1470 was divided among a colonel, 6 captains, and 116 privates, the shares of a colonel, a captain, and a private being in the proportion of 7 : 4 : 1. What did each receive?

(18) In a partnership A contributes £340 for 4 months, B £510 for 8 months, and C £850 for 10 months. They gain £270, 13s. 4d. How much does each receive?

(19) 927 policemen are to be distributed among three towns in proportion to the population, viz.: 19654; 17928; 30695. How many policemen will be sent to each town?

(20) A sum of £4000 is to be raised by 3 towns in proportion to their rentals, which are £192005, £56204, and £40561 respectively. Find the contribution of each town to the nearest pound.

SIMPLE INTEREST.

Interest is money paid for the loan of money.

The sum lent is called the **principal.**

The principal and the interest added together constitute the **amount.**

From the definitions given of interest, principal, and amount, it is evident that if any two of these sums are known, the third can readily be found either by addition or by subtraction.

Interest is usually reckoned at so much per cent. per annum —that is, at so much for £100 for a year. (In the wording of questions the phrase 'per annum' is frequently omitted.)

The following are examples of questions in interest:

I. To find the simple interest.

(1) *Find the simple interest on* £560 *for* 3 *years at* 4 *per cent. per annum.*

We know that the interest on £100 for 1 year = £4.

$\therefore$ " " £1 " 1 " = £$\frac{4}{100}$.

$\therefore$ " " £560 " 1 " = £$\frac{4 \times 560}{100}$

$\therefore$ " " £560 " 3 years = £$\frac{4 \times 560 \times 3}{100}$

= £67, 4s.

From the above we can deduce the rule, 'To find the simple interest, multiply the principal by the rate per cent. and by the number of years, and divide by 100.'

(Months should be expressed as fractions of a year.)

This is expressed by the formula—

$$I = P \times \frac{r \times t}{100},$$

where I = interest, P = principal, r = rate per cent., and t the number of years.

A little practice will enable the student to determine the quickest way of finding the value of the expression $P \times \frac{r \times t}{100}$.

EXAMPLES.—(1) *Find the simple interest on* £810, 15s. 6d. *for* 3 *years and* 4 *months at* $2\frac{1}{2}$ *per cent. per annum.*

$$\frac{r \times t}{100} = \frac{2\frac{1}{2} \times 3\frac{1}{3}}{100} = \frac{5}{2} \times \frac{10}{3} \times \frac{1}{100} = \frac{1}{12};$$

$\therefore$ I = $\frac{1}{12}$ of £810, 15s. 6d. = £67, 11s. $3\frac{1}{2}$d.

(2) *Find the simple interest on* £742, 6s. 7½*d. for* 2 *years and* 7 *months at* $1\frac{3}{4}$ *per cent.*

$$\frac{r \times t}{100} = \frac{1\frac{3}{4} \times 2\frac{7}{12}}{100} = \frac{7}{4} \times \frac{31}{12} \times \frac{1}{100} = \cdot 045208 \ldots$$

P = £742·33125.

Multiply by the contracted method, getting the answer correct to 3 decimal places.

$$\begin{array}{r|l} 7{\cdot}4233 & \\ 4{\cdot}5208 & \\ \hline 29{\cdot}6932 & \\ 3{\cdot}7117 & \\ 1485 & \\ 59 & \end{array}$$

I = £33·559 = £33, 11s. 2¼d.

All determinations of simple interest are simply questions in percentages, and the student thoroughly familiar with the table on page 81 will often be able to find the answer mentally.

NOTE.—5 % per annum = 1s. per £ per year = 1d. per month = ¼d. per week (approximately).

To equalise the time to a 5 per cent. rate. We can reduce the problem of finding simple interest at any rate per cent. for a given time to the question of finding the interest at 5 per cent. for an equivalent time. For example, If a sum of money is invested for 4 years at r per cent., for what time must the same sum be invested at 5 per cent. to produce the same interest? Evidently

$$4 \times \frac{r}{5} \text{ years or } 4 \times \frac{2r}{10} \text{ years.}$$

Rule.—To equalise the time to a 5 per cent. rate, multiply the given time by double the rate and divide by 10.

EXAMPLES.—(1) *Find the simple interest on* £320 *for* 2 *years at* $3\frac{1}{2}$ *per cent.*

I = interest on £320 for 2 years at $3\frac{1}{2}$ per cent.
= " " £320 " $\frac{14}{10}$ " 5 "
= 5 per cent. of £320 × $\frac{14}{10}$.
= 5 per cent. of £448 = 448s. = £22, 8s.

(2) *Find simple interest on* £180 *for* 3 *years and* 2 *months at* $1\frac{1}{2}$ *per cent.*

I = interest on £180 for 3 years and 2 months at $1\frac{1}{2}$ per cent.
= " " £180 " $\frac{3}{10}$ " " $\frac{6}{10}$ " 5 "
= 5 per cent. on £162 for 1 year + 5 per cent. on £108 for one month.
= 162s. + 108d. = 171s. = £8, 11s.

EXERCISE 46. (*Mental.*)

Find the simple interest on:

(1) £300 for 1 year at 4 per cent.
(2) £480 " 1 " " $2\frac{1}{2}$ "
(3) £867 " 1 " " 5 "
(4) £293 " 1 " " $1\frac{1}{2}$ "
(5) £380 " 2 years " 5 "
(6) £720 " 2 " " $2\frac{1}{2}$ "
(7) £400 " 3 " " 4 "
(8) £120 " 4 " " $3\frac{1}{2}$ "
(9) £229 " 1 month " 5 "
(10) £540 " 1 " " $2\frac{1}{2}$ "
(11) £329 " 1 year and 1 month at 5 per cent.

(12) £642 for 1 year and 1 month at $2\frac{1}{2}$ per cent.
(13) £200 " 2 years and 3 months " 5 "
(14) £128 " 1 year " $7\frac{1}{2}$ "
(15) £1000 " 1 " " $3\frac{1}{2}$ "
(16) £720 " 1 month " $4\frac{1}{2}$ "
(17) £246 " 1 week " 5 "
(18) £120 " 3 years and 2 months " $1\frac{1}{2}$ "
(19) £540 " 4 " " 1 month " 5 "
(20) £150 " 2 " " 4 months " $2\frac{1}{2}$ "

EXERCISE 47.

Calculate the simple interest, correct to the nearest penny, on:

(1) £625 for 4 years at $4\frac{1}{2}$ per cent.
(2) £560 " $2\frac{1}{4}$ " $4\frac{1}{2}$ "
(3) £500 " $8\frac{1}{4}$ " $1\frac{1}{2}$ "
(4) £236, 6s. 8d. " 3 " $2\frac{1}{2}$ "
(5) £2833, 6s. 8d. " $4\frac{1}{2}$ " 3 "
(6) £4075, 10s. " $1\frac{1}{2}$ " 4 "
(7) £569, 3s. 4d. " 4 " $2\frac{1}{2}$ "
(8) £480 " 3 yr. 3 mo. at £4, 3s. 4d. per cent.
(9) £2733, 6s. 8d. " 3 " 9 " £2, 5s. "
(10) £281, 12s. " 4 " 2 " £3, 2s. 6d. "
(11) £3256 " 73 days at 3 per cent. per annum.
(12) £472, 14s. 8d. " 292 " 5 " "
(13) £8250, 10s. " 438 " $2\frac{1}{2}$ " "
(14) £5175, 15s. " 949 " $4\frac{1}{4}$ " "

INTEREST FOR A GIVEN NUMBER OF DAYS.

If, as is frequently the case, the time is given in days, express the days as a fraction of an ordinary year.

For example, *Find the interest on* £369 *for* 87 *days at* 3 *per cent. per annum.*

We can express 3 per cent. as $\frac{6}{2}$ per cent. Using this expression, the interest

$$= \frac{£369 \times \frac{6}{2} \times \frac{87}{365}}{100}$$

$$= \frac{£369 \times 6 \times 87}{2 \times 100 \times 365}$$

$$= \frac{£369 \times 6 \times 87}{73000}.$$

Hence the rule may be given, 'To find the interest for a number of days, multiply the principal by the number of days and by twice the rate per cent., and divide by 73000.'

Short Method of dividing by 73000.—There is a method of dividing by 73000, known as the 'Third, tenth, and tenth' method, which is shorter than the ordinary one, and which brings a result sufficiently accurate for practical purposes.

THIRD, TENTH, AND TENTH RULE.

Suppose we wish to divide £985963, 15s. 9d. by 73000.

We can neglect the shillings and pence, for the $\frac{1}{73000}$ part of any sum less than £1 must be less than the 73rd part of a thousandth of £1; that is, it must be less than the 73rd part of something which is less than a farthing.

To find $\frac{1}{73000}$ of £985963, divide the amount by 3, that quotient by 10, and that again by 10. Add together the four amounts thus obtained, divide the sum by 100000 (that is, point off five decimal places), and the result will approximate to the quotient required. (The proof of the rule will be given presently.)

£985963

$328654\frac{1}{3}$ = $\frac{1}{3}$ of 985963

$32865\frac{1}{2}$ = $\frac{1}{10}$ of $328654\frac{1}{3}$ (approximately)

$3286\frac{1}{2}$ = $\frac{1}{10}$ of $32865\frac{1}{2}$ "

1350769 = the sum of the four amounts.

Divided by 100000, this equals £13·50769 = £13·508 = £13, 10s. 2d.

Allowance to be made.—This method of dividing by 73000 brings an answer which is not quite correct, and we must now see what allowance must be made for the error.

The interest on £1000 for 73 days at 5 per cent.

$$= \frac{£1000 \times 5 \times \frac{1}{5}}{100} = £10.$$

By the rule generally used when 'days' are involved, the interest

$$= \frac{£1000 \times 10 \times 73}{73000}$$
$$= £730000 \div 73000.$$

Employing the 'Third, tenth, and tenth rule,' we get

$$\begin{array}{r} 730000 \\ 243333\frac{1}{3} \\ 24333\frac{1}{3} \\ 2433\frac{1}{3} \\ \hline 1000100 \end{array}$$

= (when we mark off five decimal places) £10·001 = £10 $\frac{1}{1000}$ = £10, and nearly a farthing more.

It is evident that by this rule the answer obtained is nearly a farthing too large for every £10 of interest. Hence we subtract a farthing for every approximate £10 in our answer; that is, the answer, £13, 10s. 2d., found above, should be given as £13, 10s. $1\frac{3}{4}$d.

Proof of the Rule.—Division by 73000 = multiplication by $\frac{1}{73000}$.

Now $\frac{1}{73000} = ·0000\dot{1}369863 = ·0000137$ (nearly) = $\frac{137}{10000000}$ = (dividing both numerator and denominator by 100) $\frac{1·37}{100000}$.

Hence to divide by 73000 we may multiply by 1·37 and divide by 100000—that is, point off 5 decimal places.

But when we take a 'third, a tenth, and a tenth,' and add the four quantities together, we are practically multiplying by 1·37,

$$\text{for } 1{,}37 = 1 + ·33\frac{1}{3} + ·03\frac{2}{3}$$
$$= 1 + ·33\frac{1}{3} + ·03\frac{1}{3} + ·00\frac{1}{3},$$

where $·33\frac{1}{3} = \frac{1}{3}$ of 1, and $·03\frac{1}{3} = \frac{1}{10}$ of $·33\frac{1}{3}$, and $·00\frac{1}{3} = \frac{1}{10}$ of $·03\frac{1}{3}$.

In calculating the number of days between one date and a subsequent one, it is usual to omit the first date and to include the second. Thus from 3rd May to 12th July there are 28 days (in May) + 30 days (in June) + 12 days (in July)

= 70 days. We may use a table of days for this purpose, the number of days being obtained thus.

Number opposite 12th July = 193
" " 3rd May = 123
∴ Number of days from 3rd May to 12th July = 70

TABLE OF DAYS.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29	—	88	119	149	180	210	241	272	302	333	363
30	30	—	89	120	150	181	211	242	273	303	334	364
31	31	—	90	—	151	—	212	243	—	304	—	365

In leap year add one day more if the date comes after 28th February.

The method of approximation given above can be applied to finding the total interest on *different* principals for *different* numbers of days, the interest being at the *same* rate per cent. in each case. To do this, multiply each principal by its own number of days, and find the sum of the products. Multiply this sum by twice the rate per cent., and divide by 73000.

For example, *A man deposited in the bank* £260 *on 3rd February, and* £400 *on 31st July. He withdrew* £150 *on 15th May, and* £200 *on 20th September. What interest would be owing to him at the close of the year, the rate being 3 per cent. per annum?*

He had £260 in the bank from February 3 to May 15 = 101 days.
" £110 " " May 15 to July 31 = 77 "
" £510 " " July 31 to Sept. 20 = 51 "
" £310 " " Sept. 20 to Dec. 31 = 102 "

Therefore, interest due

$$= \frac{£\{(260\times 101)+(110\times 77)+(510\times 51)+(310\times 102)\}\times 6}{73000}$$

$$= \frac{(£26260+8470+26010+31620)\times 6}{73000}$$

$$= \frac{£92360\times 6}{73000} \qquad = \frac{£554160}{73000}.$$

Applying the 'Third, tenth, and tenth' rule, we get £7·59199 = £7·592 = £7, 11s. 10d. Making allowance for the error made in dividing by 73000, the interest due = £7, 11s. 9¾d.

EXERCISE 48.

Calculate the simple interest, correct to the nearest penny, on:

(1) £368, 3s. 4d. for 560 days at 3¾ per cent. per annum.
(2) £9125, 5s. " 394 " 1⅛ " "

			per annum.
(3)	£685	from January 1 to July 1, 1893	at 4 per cent.
(4)	£1375, 10s.	" January 15 to April 30, 1894	" 5 "
(5)	£2960, 15s. 6d.	" April 1 to October 1	" 2½ "
(6)	£825, 17s. 6d.	" February 10 to Dec. 25, 1895	" 4¼ "
(7)	£1284, 6s. 8d.	" June 1, 1892, to Mar. 20, 1893	" 3½ "
(8)	£9750, 15s.	" Dec. 15, 1893, to Nov. 15, 1894	" 5¾ "
(9)	£1728	" February 1 to August 1, 1894	" 3 "
(10)	£2125, 15s.	" May 10 to October 11	" 2½ "

			per annum.
(11)	£1680, 13s. 4d.	from June 5 to December 12	at $4\frac{3}{4}$ per cent.
(12)	£2972, 16s. 8d.	" July 20, 1891, to Mar. 5, 1892	" $5\frac{1}{4}$ "
(13)	£3025, 12s. 6d.	" September 1 to February 15	" $2\frac{3}{4}$ "
(14)	£1875, 10s.	" Oct. 13, 1894, to Aug. 1, 1895	" $1\frac{5}{8}$ "

(15) A man deposited in the bank £900 on 1st January, and £650 on 1st July. He withdrew £420 on 15th March, and £370 on 31st August. If interest were allowed at the rate of 2 per cent. per annum, what interest would be due to him at the close of the year?

(16) From the following data extracted from a bank-book, calculate the interest due at the end of the year at $2\frac{1}{2}$ per cent. per annum:

January 15,	deposited £150.	February 28,	withdrew £80.
April 1,	" £120.	July 3,	" £40.
September 8,	" £70.	December 1,	" £50.

II. To find the amount.

Find the interest, and add it to the principal.

Exercise 49.

Calculate the amount, correct to the nearest penny, of:

(1)	£2000	for 3 years	at $7\frac{1}{2}$ per cent.
(2)	377, 13s.	" 1 yr. 3 mo.	" 5 "
(3)	£291, 13s. 4d.	" 6 years	" $3\frac{1}{2}$ "
(4)	£41, 13s. 4d.	" 8 months	" $4\frac{1}{2}$ "
(5)	£3050, 10s. 6d.	" 3 yr. 3 mo.	" 5 "
(6)	£222, 12s. 11d.	" 700 days	" 2s. per cent. per week.

III. To find the time.

For example, *In what time will* £450 *yield* £54 *at the rate of* 4 *per cent. per annum?*

$$I = \frac{P \times t \times r}{100} \quad \therefore \quad t = \frac{100 \times I}{P \times r} = \frac{\overset{2}{\cancel{100}} \times \overset{6}{\cancel{54}}}{\underset{5}{\cancel{450}} \times 4} = 3 \text{ years.}$$

If the question were put in this form, 'In what time will £450 amount to £504 at the rate of 4 per cent. per annum?' we should find (by subtracting) the interest gained, and proceed as above.

Exercise 50.

In what time will

(1) £750 yield £45 of interest at 3 per cent.?
(2) £320 " £48 " " 5 "
(3) £420 " £67, 4s. " " 4 "
(4) £640 " £4 " " 2½ "
(5) £875 amount to £918, 15s. at 2 per cent.?
(6) £1275 " £1549, 11s. " 3⅘ "
(7) £450 " £564, 3s. 9d. " 3½ "
(8) £1955 " £2129, 14s. 6⅔d. " 2⅔ "
(9) £27512, 10s. " £31845, 14s. 4½d. " 4½ "
(10) £250 double itself " 2½ "

IV. To find the rate per cent.

For example, *At what rate per cent. will* £203 *produce* £25, 7*s.* 6*d. as interest in* 5 *years?*

$$I=\frac{P\times t\times r}{100} \quad \therefore \quad r=\frac{100\times I}{P\times t}=\frac{100\times 25\frac{3}{8}}{203\times 5}=\frac{\overset{20}{\cancel{100}}}{1}\times\frac{\overset{25}{\cancel{203}}}{\underset{2}{\cancel{8}}}\times\frac{1}{\underset{1}{\cancel{203}}}\times\frac{1}{5}=2\tfrac{1}{2}\ \%.$$

Exercise 51.

At what rate per cent. will

(1) £500 yield £67, 10s. of interest in 1½ years?
(2) £750 " £60 " " 2 "
(3) £1250 " £159, 7s. 6d. " " 3 "
(4) £1045 " £219, 9s. " " 6 "
(5) £2575, 7s. 6d. " £193, 3s. 0¾d. " " 2½ "
(6) £225 amount to £256, 10s. in 4 years?
(7) £19, 4s. " £22, 16s. " 3 "
(8) £862, 10s. " £1037, 3s. 1½d. " 4½ "
(9) £925, 16s. 8d. " £1152, 13s. 3d. " 3½ "
(10) A sum of money double itself " 33⅓ "

V. To find the principal, having given the interest.

For example, *What sum of money will produce* £15, 12*s.* 6*d. as interest in* 2 *years at* 1¼ *per cent.?*

$$I=\frac{P\times t\times r}{100} \quad \therefore \quad P=\frac{100\times I}{r\times t}=\frac{100\times 15\frac{5}{8}}{1\frac{1}{4}\times 2}=\frac{100}{1}\times\frac{\overset{25}{\cancel{125}}}{\underset{2}{\cancel{8}}}\times\frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{5}}}\times\frac{1}{2}=£625.$$

VI. To find the principal, having given the amount.

For example, *What sum of money will amount to* £806, 5*s.* *in* 3 *years at* $2\frac{1}{2}$ *per cent. per annum.?*

£100 would amount to £$107\frac{1}{2}$ in 3 years.
Or, £$107\frac{1}{2}$ would be the amount in 3 years from £100.
$\therefore$ £1 " " " " $100 \div 107\frac{1}{2}$.
$\therefore$ £$806\frac{1}{4}$ " " " " $100 \div 107\frac{1}{2} \times 806\frac{1}{4}$
$= £750$.

Exercise 52.

What principal will yield

(1) £350 interest in 10 years at 5 per cent.?
(2) £63, 7s. " 4 " $3\frac{1}{2}$ "
(3) £112, 10s. " 4 " $4\frac{1}{2}$ "
(4) £37, 5s. " $2\frac{1}{2}$ " 3 "
(5) £13, 12s. $8\frac{1}{2}$d. " $3\frac{1}{2}$ " $4\frac{1}{4}$ "
(6) £20, 16s. 6d. " 3 " $3\frac{1}{2}$ "

What principal will amount to

(7) £500, 10s. in 5 years at 2 per cent.?
(8) £822, 10s. " 5 " $3\frac{1}{2}$ "
(9) £791, 1s. " $2\frac{5}{12}$ " $4\frac{1}{12}$ "
(10) £4022, 6s. 3d. " $3\frac{1}{2}$ " $4\frac{1}{2}$ "
(11) £1751, 16s. $10\frac{1}{2}$d. " $3\frac{1}{2}$ " $4\frac{1}{2}$ "
(12) £725, 9s. $2\frac{1}{4}$d. " $2\frac{1}{4}$ " $3\frac{1}{2}$ "

(13) What principal will give the same interest in 4 months at 3 per cent. as £312, 10s. will give in 8 months at $4\frac{1}{2}$ per cent.?
(14) A sum of money put out for 10 years at $3\frac{1}{2}$ per cent. amounts to £425, 19s. $4\frac{4}{5}$d. What was the sum?
(15) What principal invested at 6 per cent. is required to yield two scholarships (which are payable yearly), one of £35, the other of £17, 10s.?
(16) A certain sum of money gains £131, 5s. in 4 years at $3\frac{3}{4}$ per cent. Find the sum, and find also what it would gain in double the time at two-thirds of the rate.
(17) A sum of £850 amounts to £939, 5s. in 3 years. How much will it amount to in 5 years at the same rate per cent.?
(18) A sum of money was deposited at $1\frac{3}{4}$ per cent. on 28th March. When principal and interest were withdrawn on 14th January of the subsequent year, the depositor received £228, 3s. What was the sum deposited?

BANK CALCULATION OF INTEREST.

Bankers and accountants use tables to facilitate the calculation of interest. Laurie's Tables give the interest on any sum of money from £1 to £20000 at $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, and 5 per cent. per annum for 1 day, 2 days, &c., up to 365 days; for 1 month, 2 months, &c., up to 12 months; and for 1 year, 2 years, &c., up to 12 years.

Current Accounts.—Merchants and others usually keep a current account with a bank; that is, they pay in or draw out from day to day such sums as they require in their business. Except by special arrangement most British banks now allow no interest on money paid into a current account; but they charge interest on an overdraft or loan. A merchant may be allowed to overdraw his account by handing to the bank sufficient security to cover the advance. The method of calculating the interest is illustrated on the opposite page.

In the money column to the left is entered all the money which Smith gets from the bank; in the second money column all the money he pays into the bank; and in the next column is entered his total indebtedness to the bank, or the bank's total indebtedness to him, at the given date. In this case he evidently owes the bank interest on £748 for 17 days, on £1044 for 13 days, on £544 for 38 days, on £194 for 40 days, and on £244 for 23 days. The interest on £748 for 17 days = the interest on £748 × 17 for 1 day. We therefore multiply these two factors and place the result in the next column. Similarly, the interest on £1044 for 13 days = the interest on £13572 for 1 day, and so on. On 15th May the account is closed by Smith paying in £244.

The interest due = interest on £60332 for 1 day at 4 %.
= £6, 12s. 3d. (from Tables.)

Notes.—(1) This is called the method of 'extending the decimal.' The name is misleading, as there is no decimal. You simply

EXTRACT FROM BANK LEDGER.

JOHN SMITH, MERCHANT, 16 High Street, Edinburgh.

Date.		Dr.		Cr.			Balance.			Days.	Decimal.	Rate.	Interest.
1910.		£	s. d.	£	s. d.		£	s. d.					£ s. d.
Jan. 4	To cash	748	0 0			Dr.	748	0 0		17	12716		
" 21	" "	296	0 0			Dr.	1044	0 0		13	13572		
Feb. 3	By "			500	0 0	Dr.	544	0 0		38	20672	4 %	
Mar. 13	" "			350	0 0	Dr.	194	0 0		40	7760		
Apr. 22	To "	50	0 0			Dr.	244	0 0		23	5612		
											60332		6 12 3
May 15	By "			244	0 0								

multiply the amount by the number of days and get the equivalent amount for 1 day.

(2) Should the rate of interest change, the interest due at 4 per cent. is calculated and put down in the interest column. The interest at the new rate is calculated in the same way by beginning the decimal over again.

(3) Students who have not access to interest tables can calculate the interest by the third, tenth, and tenth rule.

(4) When shillings and pence are given the amount should be taken to the nearest pound.

How to find the Interest on a Deposit Receipt.—This is calculated with the aid of a decimal book. Shaw's is commonly used. The interest on deposit receipts varies with the Bank of England rate, which changes from time to time. In Shaw's Tables the time has been equalised to a 5 per cent. rate by the ordinary rule—'Double the rate and divide by 10' every day since 1st January 1860. The decimal has been added on each day. In October 1909 they are given thus:

Sept.	30,	1 per cent.	8314·1	Oct.	17,	2½ per cent.	8319·1
Oct.	1,	"	8314·3	"	18,	"	8319·6
"	2,	"	8314·5	"	19,	"	8320·1
"	3,	"	8314·7	"	20	"	8320·6
"	4,	"	8314·9	"	21,	3½ per cent.	8321·1
"	5,	"	8315·1	"	22,	"	8321·8
"	6,	"	8315·3	"	23,	"	8322·5
"	7,	1½ per cent.	8315·5	"	24,	"	8323·2
"	8,	"	8315·8	"	25,	"	8323·9
"	9,	"	8316·1	"	26,	"	8324·6
"	10,	"	8316·4	"	27,	"	8325·3
"	11,	"	8316·7	"	28,	"	8326·0
"	12,	"	8317·0	"	29,	"	8326·7
"	13,	"	8317·3	"	30,	"	8327·4
"	14,	2½ per cent.	8317·6	"	31,	"	8328·1
"	15,	"	8318·1	Nov.	1,	"	8328·8
"	16,	"	8318·6				

The interest on £1 per day at 2 per cent. = the interest on £1 for $\frac{4}{10}$ days at 5 per cent. Therefore, for every day while the

rate remains at 2 per cent. the equivalent time at 5 per cent. is ·4 days. We add this daily to the decimal. When the rate changes to 2½ per cent. we add ·5 daily to the decimal; when it changes to 3½ per cent. we add ·7 daily, and so on.

EXAMPLE.—*Calculate the interest due on a deposit receipt for £200 from 30th Sept. to 1st Nov.* 1909.

Decimal opposite 1st Nov. = 8328·8
" " 30th Sept. = 8314·1
Difference = 14·7

This is the equivalent time at 5 per cent. Therefore, look up interest table. Interest on £200 for 14·7 days at 5 per cent. = 8s. 0¾d. Or use third, tenth, and tenth rule.

$$\text{Interest} = \frac{£200 \times 14{\cdot}7 \times 10}{73000} = 29400$$
9800
980
98
£·40278 = 8s. 0¾d.

The student can quite easily construct a decimal table for any required period. Suppose the Bank of England rate is 2½ per cent. on 1st January, and that it changes to 3 per cent. on 5th January, the decimal table will begin thus:

Jan. 1,	2½ per cent.	0	Jan. 4,	2½ per cent.	1·5
" 2,	"	0·5	" 5,	3 per cent.	2
" 3,	"	1·0	" 6,	"	2·6, &c.

EXERCISE 53.

Make a decimal table to be used in calculating the interest on deposit receipts from 1st January to 30th April 1910, when the changes in the Bank of England rate were as follows: 1st January, 2 °/₀; 14th January, 2½ °/₀; 4th February, 3 °/₀; 24th February, 3½ °/₀; 16th March, 3 °/₀; 20th March, 2½ °/₀; 15th April, 2 °/₀. Use the table to find the interest on the following deposit receipts:

(1) £40 deposited on January 1, withdrawn on April 30.
(2) £100 " January 4, " March 21.
(3) £25 " January 6, " April 13.
(4) £60 " January 10, " February 28.
(5) £150 " January 2, " April 23.
(6) £250 " January 8, " April 26.
(7) £420 " January 2, " March 31.

(8) *Use the method of extending the decimal to find the interest due on the following accounts, to the nearest penny.*

(*a*)

Date.		Dr.			Cr.				Balance.			Days.	Decimal.	Rate.	Interest.		
		£	*s.*	*d.*	£	*s.*	*d.*		£	*s.*	*d.*				£	*s.*	*d.*
Jan. 2	To cash	750	0	0													
" 26	" "	200	0	0													
Feb. 24	By "				500	0	0							4%			
Mar. 16	To "	140	0	0.													
Apr. 20	By "				300	0	0										
May 15	" "				290	0	0										

(*b*)

Date.		Dr.			Cr.				Balance.			Days.	Decimal.	Rate.	Interest.		
		£	*s.*	*d.*	£	*s.*	*d.*		£	*s.*	*d.*				£	*s.*	*d.*
June 3	To cash	423	17	6													
July 21	" "	324	5	0													
Aug. 4	By "				500	0	0							5%			
Sept. 16	To "	129	14	0													
Oct. 23	By "				150	4	6										
Nov. 11	" "				227	12	0										

(*c*)

Date.		Dr.	Cr.		Balance.	Days.	Decimal.	Rate.	Interest.
		£ s. d.	£ s. d.		£ s. d.				£ s. d.
Jan. 3	To cash	450 0 0							
" 23	" "	240 0 0							
Feb. 16	" "	340 7 6							
Mar. 24	By "		400 0 0					4½ %	
Apr. 26	" "		300 0 0						
May 19	To "	200 0 0							
June 3	" "	150 0 0							
" 30	By "		680 7 6						

(*d*)

Date.		Dr.	Cr.		Balance.	Days.	Decimal.	Rate.	Interest.
		£ s. d.	£ s. d.		£ s. d.				£ s. d.
Jan. 2	To cash	427 13 4							
Feb. 28	" "	127 5 2							
Apr. 10	" "	322 17 6							
June 17	By "		430 7 6					4 %	
July 29	" "		400 0 0						
Sept. 3	To "	329 14 6							
Nov. 15	" "	120 0 0							
" 30	By "		350 0 0					5 %	
Dec. 31	" "		147 3 0						

The rate of interest changed on November 11th from 4 % to 5 %.

MISCELLANEOUS EXAMPLES IN SIMPLE INTEREST.

Exercise 54.

(1) In what time will £500 amount to £515 at $2\frac{2}{5}$ per cent.?

(2) In what time will £820, 10s. amount to £869, 14s. 7·2d. at $5\frac{1}{4}$ per cent.?

(3) At what rate per cent. will £825 amount to £866, 5s. in $3\frac{3}{4}$ years?

(4) Find the interest, true to the nearest penny, on £680, 12s. 6d. from 2nd November to 7th December at $3\frac{1}{2}$ per cent. per annum.

(5) What sum put out to interest for $3\frac{1}{3}$ years, at $1\frac{1}{2}$ per cent., will amount to £980?

(6) What principal will gain £7, 15s. $6\frac{2}{3}$d. in ten months at 8 per cent. per annum?

(7) At what rate per cent. will £350, 12s. 6d. amount to £371, 13s. 3d. in $2\frac{1}{2}$ years?

(8) Find the interest, true to the nearest penny, on £1425, 7s. 6d. from 18th March to 16th July at $2\frac{1}{2}$ per cent. per annum.

(9) In what time will £1000 gain £87, 10s. as interest at 3·75 per cent.?

(10) What sum of money, put out to interest at 2·8 per cent., will gain £8, 9s. 2d. in 3·625 years?

(11) In what time will £65·5 amount to £79·91 at 5·5 per cent.?

(12) Find the interest, correct to the nearest penny, on £875 from 8th June to 22nd August at $4\frac{8}{9}$ per cent.

(13) What principal will amount to £1160, 10s. in 1·1 year at 5 per cent.?

(14) At what rate per cent. per annum will £380 gain £7, 12s. as interest in 10 months?

(15) Find the interest, correct to the nearest penny, on £3025, 17s. 6d. from 6th November 1893 to 7th July 1894 at $4\frac{1}{2}$ per cent. per annum.

(16) At what rate per cent., simple interest, will £1560 double itself in 20 years?

(17) I deposited £420 on 5th May in a bank paying interest at 3 per cent. per annum. On June 4th the interest fell to $2\frac{1}{2}$ per cent., and on 10th September it rose to $2\frac{3}{4}$ per cent. Find, to the nearest penny, the interest due on 12th November in the same year.

(18) A father, who had given his eldest son £3000 on the 23rd June 1909, and his second son £1000 on the 23rd December 1909,

died in April 1910, leaving his estate to be divided equally among his three sons, the sums paid to his sons during his lifetime, along with simple interest at 5 per cent. up to the date of final settlement, to be considered as part payment of their shares. The remainder of his estate was found to be £6734, and the whole was divided on 23rd June 1910. Find what share of the £6734 each son received.

COMPOUND INTEREST (WITHOUT LOGARITHMS).

When money is lent at simple interest, the interest is paid to the lender at the end of each year, or other fixed period. When money is lent at compound interest, the interest is not paid to the lender periodically, but is added to the principal at the end of each year, or other fixed period; and the interest for the next year, or period, is calculated on this increased principal.

Interest = Final Amount − Principal.

EXAMPLE.—*Find the compound interest on* £200 *for* 3 *years at* $4\frac{1}{2}$ *per cent. per annum.*

	£	
	200	= Principal.
4 % of £200	= 8	= Interest for 1st year.
½ % "	= 1	
	209	= Amount at end of 1st year.
4 % of £209	= 8·36	= Interest for 2nd year.
½ % "	= 1·045	
	218·405	= Amount at end of 2nd year.
4 % of £218·405	= 8·7362	= Interest for 3rd year.
½ % "	= 1·0920	
	228·2332	= Amount at end of 3rd year.

Interest = £228, 4s. 8d. − £200 = £28, 4s. 8d.

NOTE.—Always use decimals, and obtain a result correct to three decimal places.

This method becomes tedious when we require the compound interest for a lengthened period, and bankers and accountants use tables to shorten the work.

Suppose £ P is invested at r °/₀ for a number of years, and that A_1, A_2, A_3, &c., denote the amounts at the end of 1 year, 2 years, 3 years, &c., then

$$A_1 = P + \frac{r}{100} \text{ of } P = \frac{100\,P + rP}{100} = P\left(\frac{100+r}{100}\right)$$

$$A_2 = A_1 + \frac{r}{100} \text{ of } A_1 = A_1\left(\frac{100+r}{100}\right) = P\left(\frac{100+r}{100}\right)^2$$

$$A_3 = A_2 + \frac{r}{100} \text{ of } A_2 = A_2\left(\frac{100+r}{100}\right) = P\left(\frac{100+r}{100}\right)^3$$

. . . . . .

$$A_n = P\left(\frac{100+r}{100}\right)^n$$

In the above example, P = £200, and $r = 4\frac{1}{2}$, so that $\frac{100+r}{100}$ $= 1{\cdot}045$; $\therefore A_3 = £200 \times 1{\cdot}045^3 = £200 \times 1{\cdot}14117 = £228{\cdot}234$.

Tables have been constructed giving the value of the fraction $\left(\frac{100+r}{100}\right)^n$ for all commonly occurring rates.

To find the Principal when the Amount is given:

For example, *Find what sum will amount to* £1000 *in* 7 *years at* $3\frac{1}{2}$ °/₀.

From above, £1000 = Principal × $1{\cdot}035^7$;

$\therefore$ Principal = $£\frac{1000}{1{\cdot}035^7} = £\frac{1000}{1{\cdot}27227} = £785{\cdot}997 =$ £785, 19s. 11¼d.

£785, 19s. 11¼d. is called the **Present Value** of £1000.

If the interest is added half-yearly (or quarterly), we take half (or quarter) the rate, and twice (or four times) the number of periods.

EXAMPLE.—*Find the amount of* £300 *at the end of* 2 *years at* 5 °/₀, *the interest being added half-yearly.*

Amount = £300 × $1{\cdot}025^4$ = £300 × 1·10381 = £331·143 = £331, 2s. 10d.

If the amount is to be found for so many complete periods and a fraction of a period, we find the amount at the end of the number of complete periods given, and then find

the amount of this increased principal for the fraction of the period.

EXAMPLE.—*Find the amount of* £326, 17*s.* 6*d. for* $3\frac{1}{2}$ *years at* 5 °/₀ *per annum.*

Amount at the end of 3 years $= £326{\cdot}875 \times 1{\cdot}05^3 = £326{\cdot}875 \times 1{\cdot}15763$
$= £378{\cdot}400$

Interest for half-year $= \frac{1}{20}$ of this $= \quad 9{\cdot}460$

$\therefore$ Amount $= £387{\cdot}860 = £387,\ 17s.\ 2\frac{1}{2}d.$

EXERCISE 55.

Find, to the nearest penny, the compound interest on:

(1) £270 for 2 years at 3 per cent. per annum.
(2) £1725 " 3 " 5 " "
(3) £125 " 3 " $2\frac{1}{2}$ " "
(4) £500 " 4 " 5 " "
(5) £6271, 6s. 6d. " 2 " $3\frac{3}{4}$ " "
(6) £1875, 15s. " 3 " 4 " "
(7) £2371, 13s. 8d. " 2 " $5\frac{1}{4}$ " "
(8) £4500 " $1\frac{1}{2}$ " 3 " "
(9) £4250 " $2\frac{1}{2}$ " 5 " "
(10) £104 " $1\frac{3}{4}$ " 5 " "

(11) What sum will amount to £926, 2s. in 3 years at 5 °/₀?

(12) What sum will amount to £1000 in 3 years at 6 °/₀ (to nearest shilling)?

(13) What sum must be invested now at 5 °/₀ so as to be worth £810, 6s. 9d. at the end of 4 years?

(14) A youth of 18 is to receive a legacy of £1175 when he reaches his majority (21). Find the present value at $3\frac{3}{4}$ °/₀.

(15) The population of a town increases yearly by 27 per 1000, and it is now 382,620. What will it be at the end of 5 years?

(16) Find the amount of £123 at the end of 2 years at 4 °/₀—interest paid half-yearly.

(17) Find the amount of £2000 at the end of $2\frac{1}{2}$ years at 4 °/₀—interest paid half-yearly.

(18) Find the present value of a legacy of £651, 6s. 8d. due in $2\frac{3}{4}$ years, the rate of interest being 3 °/₀.

(19) Find the difference between the simple and compound interest on £4250 for 4 years at 5 °/₀.

(20) A man saves every year £150, and invests it at the end of the year at 5 °/₀ compound interest. Find the amount of his savings at the end of the fourth year.

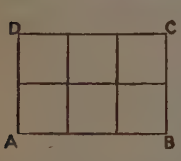
AREA OF RECTANGLES.

A **rectangle** is a four-sided figure, of which the opposite sides are parallel, and all the angles right angles.

To find the area of a rectangle. Express the length and breadth in the *same denomination* (miles, yards, feet, &c.), and multiply them together.

For example, to find the area of a rectangle 3 inches long and 2 inches broad.

Let the figure ABCD be such a rectangle, having the length AB equal to 3 inches, and the breadth BC equal to 2 inches. If we divide AB into three equal parts, and BC into two equal parts, and complete the figure as shown in the diagram, we divide the original figure into six squares, the side of each of which is exactly an inch. Hence, each of these squares is called a square inch, and it is evident that the area of the rectangle equals 6 square inches; that is, it equals the length multiplied by the breadth.



Similarly, the area of a rectangle 8 yards long and 5 feet broad = (24 × 5) square feet = 120 square feet = 13 square yards 3 square feet.

Since the area = length × breadth, it follows that the area divided by the length gives the breadth, and that the area divided by the breadth gives the length.

In measuring fields, roads, &c., what is called a 'Gunter's Chain' is used. This is exactly 22 yards long (the length of a cricket pitch), and is composed of 100 links.

The following table will be useful:

22 yards or 100 links = 1 chain.
10 chains = 1 furlong.
80 chains or 8 furlongs = 1 mile.

Suppose a certain area were exactly a chain (or 100 links) long, and the same distance wide. Evidently, we might

describe the area as a square chain, or as (22×22) square yards, or as (100×100) square links.

Therefore, a square chain = 484 square yards or 10000 square links.

The following table will be useful:

484 square yards or 10000 square links = 1 square chain.
4840 square yards or 10 square chains = 1 acre.
640 acres = 1 square mile.

EXAMPLE.—*A field is* 6 *chains* 14 *yards long, and* 4 *chains* 6 *yards wide. Find its area in acres and square yards.*

Length = 146 yd.; breadth = 94 yd.; therefore, area = 13724 sq. yd.
= 2 ac. 4044 sq. yd.—Ans.

EXERCISE 56.

(1) Find the area of a slate 8 in. long and 6 in. broad.
(2) What is the area of a blackboard 2 yd. 1 ft. long and 1 yd. 2 ft. broad?
(3) A room is 8 yd. 1 ft. long and 6 yd. 2 ft. wide. How many square yards are there in the floor?
(4) A large field is 363 yd. long and 280 yd. broad. How many acres are there in the field?
(5) If a gentleman's estate were $3\frac{1}{2}$ miles long and $2\frac{1}{4}$ miles broad, how many acres would it contain?
(6) If a bridge were three-quarters of a mile long and $13\frac{1}{3}$ yd. wide, what would be its area in square yards?
(7) Each side of a square room is 3 yd. 2 ft. 3 in. long. Find the area of the floor.
(8) Find the area of a field which is 26 po. 3 yd. long and 18 po. 5 yd. wide. (Give your answer in acres, &c.)
(9) The length of a room is 5 yd. 9 in., and its width is $4\frac{1}{2}$ yd. Find the combined areas of the floor and the ceiling.
(10) A garden-path, 4 po. 2 yd. long and 4 ft. wide, was to be covered with sand, at 1s. 3d. per sq. yd. Find the cost.
(11) Ten doors, each $2\frac{1}{3}$ yd. high and $4\frac{1}{2}$ ft. wide, were to be painted on both sides. Find the cost at $\frac{1}{2}$d. per square foot.
(12) The floor and ceiling of a room, 16 yd. 2 ft. long and 12 yd. broad, were to be painted. What would this cost at 3d. per square yard?

(13) A field is $1\frac{1}{4}$ fur. in length and 24 po. in width. If it cost for ploughing 2d. for every 100 sq. yd., find the cost of ploughing the whole field.

(14) One of the walls of a room was $13\frac{1}{2}$ ft. long and 12 ft. high. In it there was a window $6\frac{1}{2}$ ft. high and 4 ft. broad. Find the cost of painting the rest of the wall at 2d. per sq. ft.

(15) A school contains a large room $23\frac{1}{3}$ yd. by $14\frac{2}{3}$ yd., and two square class-rooms, each 25 ft. long. If 10 sq. ft. are to be allowed for each scholar, how many scholars will the whole school accommodate?

(16) A field is 25 chains long and 10 chains wide. Find the rent of the field at £2, 3s. 4d. per acre.

(17) The area of a sheet of paper is 30 sq. in., and its breadth is 6 in. Find its length.

(18) If the width of a room 192 sq. ft. in area is 12 ft., what is its length?

(19) A garden-path contains 48 sq. yd. If it is $4\frac{1}{2}$ ft. wide, what is its length?

(20) Find the breadth of a field which contains 4 ac. 3 ro., and which is 190 yd. long. (Give your answer in yards.)

(21) The area of a wall is 15 sq. yd. Find the height of the wall if its length is $13\frac{1}{2}$ ft.

(22) A field is 3 chains long, and it is $1\frac{1}{2}$ ac. in area. Find its width.

PRACTICAL APPLICATIONS.

Two rectangles of different shapes may be equal to each other in area. Thus, a long strip of narrow carpet may be equal in area to the broad floor of a room, and by cutting may be made to cover the room exactly.

(1) *A rectangle is* 9 *feet long and* 8 *feet broad. How broad must another rectangle be which is equal in area to the former and is* 12 *feet long?*

The area of the first rectangle $= (9 \times 8)$ sq. ft. $= 72$ sq. ft.

We have previously seen that the breadth of a rectangle equals the area divided by the length; therefore, the breadth of the second rectangle $= (72 \div 12)$ ft. $= 6$ ft.

Instead of multiplying out, we might have continued to

use the expression (9×8) sq. ft. In that case we should say that the breadth of the second rectangle $= \frac{9 \times 8}{12}$ ft. $= 6$ ft. As a rule this is the most convenient method of working.

(2) *What length of carpet,* $\frac{3}{4}$ *yard wide, will be needed to cover a floor* 15 *feet long and* 13 *feet* 6 *inches broad? Find the cost at* 3*s.* 6*d. per yard.*

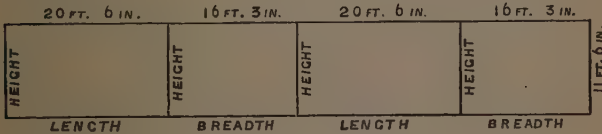
Area of floor $= (5 \times 4\frac{1}{2})$ yd.

Therefore, length of carpet $= \frac{5 \times 4\frac{1}{2}}{\frac{3}{4}}$ yd. $= 30$ yd.

Cost $= 3$s. 6d. $\times 30 = £5$, 5s.

(3) *Find the combined area of the four walls of a room which is* 20 *feet* 6 *inches long,* 16 *feet* 3 *inches broad, and* 11 *feet* 6 *inches high.*

The four walls may be supposed to be placed end to end so as to form a single rectangle, as in the accompanying diagram.



The whole area is now seen to be

$$(20 \text{ ft. } 6 \text{ in.} + 16 \text{ ft. } 3 \text{ in.}) \times 2 \times (11 \text{ ft. } 6 \text{ in.})$$
$$= (36\tfrac{3}{4} \times 2 \times 11\tfrac{1}{2}) \text{ sq. ft.} = 845\tfrac{1}{4} \text{ sq. ft.}$$
$$= 93 \text{ sq. yd. } 8 \text{ sq. ft. } 36 \text{ sq. in.}$$

The method adopted in working the above example may be expressed as follows: To find the area of the walls, find the distance round the room and multiply by the height.

The amount of paper, 21 in. wide, which would be used to paper the above room (making no allowance for doors, windows, &c.) would evidently be the area of the walls

divided by the breadth of the paper—that is, $(845\frac{1}{4} \div \frac{21}{12})$ ft. $= (\frac{3381}{4} \times \frac{12}{21})$ ft. $= 483$ ft. $= 161$ yd.

Or, the length of the paper might be expressed as $(36\frac{3}{4} \times 2 \times 11\frac{1}{2})$ sq. ft. $\div \frac{21}{12}$ ft. $= (\frac{147}{4} \times \frac{2}{1} \times \frac{23}{2} \times \frac{12}{21})$ ft. $= 483$ ft. $= 161$ yd.

Note.—In Britain wall-paper is usually 21 in. wide, and is sold by the 'piece.' A 'piece' is 12 yd. long, and therefore contains $(\frac{7}{12} \times 12)$ sq. yd. $= 7$ sq. yd. To find the number of 'pieces' required for a room, we should, therefore, find the area of the walls in square yards, and divide by 7.

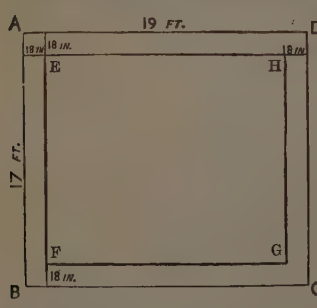
(4) *How many garden-plots, each* 22 *yards long and* 8 *yards wide, could be made out of a square field* $\frac{1}{4}$ *mile long?*

Area of field $= (440 \times 440)$ sq. yd.

Area of each plot $= (8 \times 22)$ sq. yd.

Therefore, number of plots $= \dfrac{440 \times 440}{8 \times 22} = 1100.$

(5) *A room measuring* 19 *feet by* 17 *feet is to be carpeted so as to leave an uncovered piece* 18 *inches broad next the walls all round the room. Find the area of the uncovered part.*



In the accompanying diagram ABCD represents the floor of the room, and EFGH the carpet.

The area of the room $=$ (19×17) sq. ft. $= 323$ sq. ft. As the margin left uncovered equals 18 in., it is evident that the area to be carpeted $= 16$ ft. by 14 ft. $=$ 224 sq. ft. Therefore, the area of the uncovered part $=$ 323 sq. ft. $- 224$ sq. ft. $= 99$ sq. ft. $= 11$ sq. yd.

Exercise 57.

(1) A room is 27 sq. yd. in area. How many yards of carpet, three-quarters of a yard wide, will be needed to cover it?

(2) How many yards of carpet, 2 ft. 3 in. broad, will be needed for a room $13\frac{1}{2}$ ft. long and 4 yd. broad?

(3) Find the cost of the length of carpet, three-quarters of a yard wide, needed to cover a room $5\frac{1}{2}$ yd. by $3\frac{1}{4}$ yd., if the carpet be 3s. 6d. per yd.

(4) How much paper, $\frac{1}{2}$ yd. wide, will be needed to cover the walls of a square room which is 4 yd. long and 3 yd. high?

(5) How many yards of paper, $1\frac{3}{4}$ ft. wide, will be used in papering a room 8 yd. long, 6 yd. wide, and 4 yd. high, supposing the door and window take up 63 sq. ft.?

(6) Find the cost of papering the four walls of a room 9 ft. long, 8 ft. broad, and 8 ft. high, if the paper be 2 ft. wide and cost $1\frac{1}{2}$d. per yd.

(7) A park was half a mile long and a quarter of a mile broad. If a square pond was made in the middle of it, 80 yd. long, find the area in sq. yd. of the remaining part of the park.

(8) An oblong flower-bed was 12 yd. long and 8 yd. broad. If a path $4\frac{1}{2}$ ft. wide were made all the way round the inside of the bed, find the area of the path.

(9) A grass plot is 120 ft. long and 80 ft. wide. It is surrounded by a gravel-walk 6 ft. wide. Find the area of the gravel-walk in square yards.

(10) A picture is 6 ft. by 4 ft., and outside the picture is a frame 4 in. broad. Find the cost of gilding the frame at 1s. 6d. a square foot.

(11) Find the cost of surrounding a square bowling-green, 18 yd. long, with a paved walk $1\frac{1}{2}$ yd. wide, at 5s. per square yard.

(12) Some zinc sheets are 7 ft. long and 3 ft. wide. If each of them cost 8s. 6d., what will be the price of sufficient to cover a roof 56 ft. long and 18 ft. wide?

(13) How many acres are there in a field which is a quarter of a mile long and 242 yd. wide?

(14) A room is 36 ft. long, 18 ft. broad, and 12 ft. high. Find the whole cost of plastering the walls at 1s. per sq. yd., and the ceiling at 1s. 6d. per sq. yd.

(15) Find the cost of carpeting a room, 24 ft. by 18 ft., with carpet 27 in. wide, at 5s. 4d. per yd.

(16) Find the rent paid for a field 220 yards square at £2, 14s. 6d. per acre.

(17) If a window be 8 ft. 2 in. high by 5 ft. 3 in. wide, how many panes of glass will fill it, if each pane measures 14 in. by 9 in.?

(18) In making a road 33 ft. broad, 32 acres of land were used. Find the length of the road in miles.

(19) A court, 20 yd. 2 ft. 6 in. long, by 12 yd. 2 ft. 3 in. broad, is paved with flags, each of which is 2 ft. 1 in. by 1 ft. 5 in. How many flags are needed?

(20) Find the cost of carpeting a room, 18 ft. 9 in. by 17 ft. 6 in., with carpet 27 in. wide, at 5s. 3d. per yd.

(21) Find the cost of making a road, $5\frac{1}{2}$ yd. wide and 300 chains long, at £140 per acre.

(22) A joiner had a plank 9 in. wide. What length must he cut off so as to have exactly $13\frac{1}{2}$ sq. ft.?

(23) How many yards of paper, 27 in. wide, will be required to cover the four walls of a room 37 ft. square and 10 ft. 6 in. high?

(24) The expense of paving a street half a mile long, at $7\frac{1}{2}$d. per sq. yd., was £430, 16s. 8d. Find the breadth of the street.

(25) A carpet 13 ft. 6 in. by 10 ft. 9 in. is laid on a floor 20 ft. 6 in. by 12 ft. 9 in. Find the cost of staining the rest of the floor at 3d. per sq. yd.

(26) How many tiles $4\frac{1}{2}$ in. by $2\frac{1}{2}$ in. are required for a square court each side of which measures 12 ft. 6 in.?

(27) How many planks, each $13\frac{1}{2}$ ft. long and $10\frac{1}{2}$ in. wide, will be needed for a platform 54 yd. long and 21 yd. wide? Find the cost at $5\frac{1}{2}$d. per sq. ft.

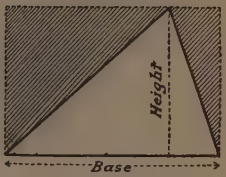
(28) Find the number of granite blocks required to pave a street 1 mile long and 16 yd. wide, each block being 4 in. broad and 12 in. long.

(29) A quadrangle is 50 ft. long by 40 ft. broad. It is crossed in each direction by a path 10 ft. broad, and the remainder has to be turfed. How many strips of turf $1\frac{1}{2}$ ft. long and 6 in. broad will be required for the purpose?

(30) Find the cost of the lead lining of a rectangular cistern 3 ft. 2 in. long, 2 ft. 8 in. wide, and 2 ft. 6 in. deep, supposing that the lead cost 3d. per lb., and that 7 lb. of lead cover a square foot. (The cistern has no roof.)

AREA OF OTHER RECTILINEAL FIGURES.

Area of a Triangle.—Area of triangle $= \frac{\text{Base} \times \text{Height.}}{2}$



This is at once evident from the given figure, for the triangle is half of the rectangle constructed on the same base and having the same height.

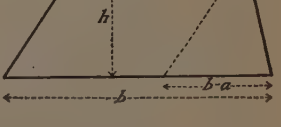
Area of a Parallelogram.—A parallelogram is a four-sided figure with its opposite sides parallel.

Area of parallelogram
= Base × Height.

In the figure the shaded portions are equal, so that the area of the parallelogram is equal to that of a rectangle having the same base and height.

Area of a Trapezium.—A trapezium is a four-sided figure with two of its opposite sides parallel.

In the figure we have divided the trapezium into a parallelogram and a triangle.

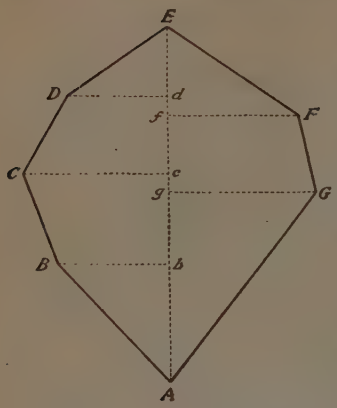


Area of parallelogram $= a \times h$;

area of triangle $= \frac{(b-a)h}{2}$;

by adding, area of trapezium $= \left(\frac{a+b}{2}\right) \times h$;

= half-sum of parallel sides × height.



Land Surveying.—Land surveyors have very often to find the area of irregular figures. Suppose *ABCDEFG* represents a field whose area is to be found. We first select one of the diagonals as base-line or chain-line, then find where each of the perpendiculars drawn from the corners to this line meets it. The measurements are entered in a field-book thus:

Chains.		
	To *E*	
	33	
To *D* 9	27	
	25	12 to *F*
To *C* 14	20	
	18	13 to *G*.
To *B* 12	11	
	From *A*	

The area of each triangle and trapezium is then calculated, and the results added.

$$\triangle ABb = \frac{11 \times 12}{2} = 66$$

$$\text{Trap. } BCcb = \frac{12+14}{2} \times 9 = 117$$

$$\text{Trap. } CDdc = \frac{14+9}{2} \times 7 = 80{\cdot}5$$

$$\triangle DEd = \frac{9 \times 6}{2} = 27$$

$$\triangle EFf = \frac{8 \times 12}{2} = 48$$

$$\text{Trap. } FGgf = \frac{12+13}{2} \times 7 = 87{\cdot}5$$

$$\triangle GAg = \frac{18 \times 13}{2} = 117$$

$\therefore$ Area of field = 543 sq. chains
= 54·3 acres.

Exercise 58.

(1) *Find the area of the following triangles:*

(*a*) Base = 25 yd.; height = 12 yd.
(*b*) " = 27 yd. 2 ft. 10 in.; height = 3 yd. 1 ft. 7 in.
(*c*) " = 24·72 m.; height = 8·75 m.
(*d*) " = 26·24 chains; height = 12·29 chains.
(*e*) " = 21 ft.; height = 9 ft.

(2) *Find the area of the following trapeziums:*

(*a*) Parallel sides = 75 yd. and 29 yd.; height = 17 yd.
(*b*) " = 24·26 m. and 18·34 m.; height = 10·25 m.
(*c*) " = 34·29 chains and 27·34 chains; height = 16·4 chains.

(3) Two triangular grass plots would, if put together, exactly form a square, having a side of 13½ yd. Find the area of each plot.

(4) The sides containing the right angle of a certain triangle measure 15 ft. and 14 ft. respectively. Find the area of the triangle.

(5) In a right-angled triangle the base measured 1 ft. 8 in. and the perpendicular 2 ft. 3 in. Find the area.

(6) How many acres are contained in a triangular field, one corner of which forms a right angle, if the sides containing the right angle are 37 chains 50 links and 16 chains respectively?

(7) A field is in the form of a right-angled triangle, the base being 18¾ po., and the perpendicular 24 po. Find the rent of the field at £2 per acre.

(8) Find the number of square yards in a triangle the base of which is 49½ ft. and the perpendicular height 42¾ ft.

(9) One side of a three-cornered field measures 17½ chains, and the perpendicular from the opposite corner to that side measures 8¼ chains. Find the area of the field.

(10) One side of a triangular courtyard measures $32\frac{1}{3}$ yd., and the perpendicular on it from the opposite angle is $20\frac{1}{3}$ yd. Find the cost of paving it at 2s. 3d. per sq. yd.

(11) Find the area of the fields from the following entries in a surveyor's field-book :

(*a*)

Chains.		
	To *D*	
	30·8	
	25·2	24·3 to *C*
To *E* 18·2	21·6	
	10·2	16·7 to *B*
	From *A*	

(*b*)

Yards.		
	To *D*	
	1000	
To *E* 246	730	
	724	180 to *C*
To *F* 124	540	
To *G* 270	325	
	300	350 to *B*
	From *A*	

(*c*)

Metres.		
	To *E*	
	1500	
To *F* 280	1160	
	1000	250 to *D*
To *G* 310	670	240 to *C*
	320	245 to *B*
To *H* 200	180	
	From *A*	

(*d*)

Chains.		
	To *E*	
	32·6	
	28·7	11·9 to *D*
To *F* 16·5	26·4	
	20·9	8·7 to *C*
	15·2	15·6 to *B*
To *G* 14·8	8·6	
	From *A*	

(12) From the following entries in a field surveyor's book calculate the rent of each of the following fields, the rent being charged on (*a*) at £2, 0s. 4d. per acre, on (*b*) at £3, 7s. 6d. per acre, on (*c*) at 145 francs per hectare, and on (*d*) at £2, 12s. 6d. per acre :

(*a*)

Yards.		
	To *B*	
	600	
To *G* 160	480	
	450	200 to *D*
To *F* 100	350	
To *E* 180	250	
	100	250 to *C*
	From *A*	

(*b*)

Chains.		
	To *B*	
	31·89	
To *F* 3·92	23·43	12·24 to *G*
To *E* 6·18	16·19	
To *D* 4·88	11·44	
To *C* 5·62	6·18	
	From *A*	

(*c*)

Metres.		
	To *B*	
	360	
	286·4	96·4 to *F*
To *C* 90·8	208·5	
	150·6	86·5 to *E*
	98·2	102·3 to *D*
	From *A*	

(*d*)

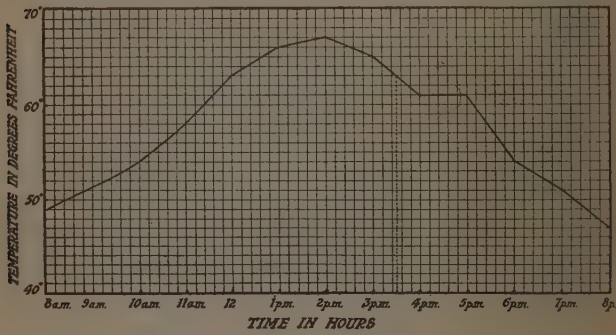
Chains.		
	To *E*	
	22·3	
	18·2	4·8 to *D*
	15	1·6 to *C*
To *F* 5·7	14·5	
	7·6	6·9 to *B*
To *G* 6·8	5·3	
	From *A*	

GRAPHS.

When a quantity changes from time to time, the change can be shown to the eye with the aid of a sheet of squared paper. Suppose the temperature of a room is given in the following table:

Hour.	8 A.M.	9 A.M.	10 A.M.	11 A.M.	12 N.	1 P.M.	2 P.M.	3 P.M.	4 P.M.	5 P.M.	6 P.M.	7 P.M.	8 P.M.
Temperature in degrees Fahrenheit.	49°	51°	54°	58°	63°	66°	67°	65°	61°	61°	54°	51°	47°

Let us take a sheet of squared paper ruled in inches and tenths of an inch. Along the lowest horizontal line mark points every half-inch, and, beginning at the left hand, indicate these points as 8 A.M., 9 A.M., &c. We shall take one-tenth of an inch on a vertical line to represent 1 degree of temperature, and as the temperature never falls below 40° we can call the first or lowest line 40°. Complete the temperature scale along the left-hand vertical line. On the 8 A.M. vertical line mark the point which indicates 49°; on the 9 A.M. vertical line mark the point which indicates 51°, and continue the process for the other hours. Then join



each pair of successive marks by straight lines. The result is a series of short straight lines which form, as a whole, an irregular curve. The curve shows you at a glance how the temperature rose and fell during the day. It is called a temperature graph. It may also be used to find the approximate temperature at any hour of the day. For example, at 3.30 P.M. the approximate temperature of the room was 63°.

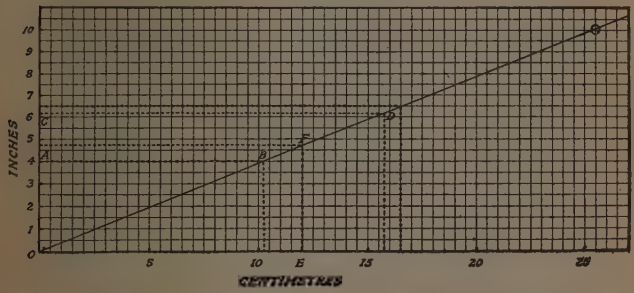
A similar method may be employed to solve certain questions in arithmetic.

EXAMPLE.—Draw a graph for converting inches into centi-

metres, taking 10 inches = 25·4 centimetres, and find from the graph the number of centimetres equal to 4 inches and to 6·2 inches, and the number of inches equal to 12 centimetres and to 16·5 centimetres.

Take the horizontal line for the centimetre scale, and let two-tenths of an inch represent 1 centimetre. Take the vertical line for the inch scale, and let two-tenths of an inch represent 1 inch.

Since 10 inches = 25·4 centimetres, we go along the centimetre scale 25·4 divisions, and then go vertically upwards till we come to the horizontal line indicating 10 inches, and mark the point where these two lines meet. Join this point

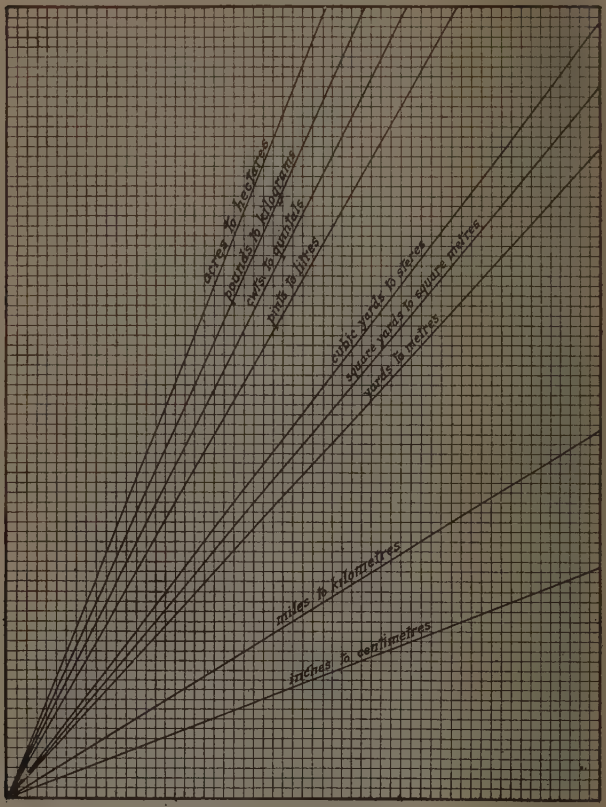


to the origin by a straight line. This straight line is the graph required, and may be produced if necessary.

Where, as in this example, a change in the one quantity gives a proportional change in the other quantity, the graph is always a straight line. All we have to do is to find two points on it, join these with the aid of a ruler, and produce the line if necessary.

To find the number of centimetres in 4 inches, we go up the inch scale till we come to the 4-inch mark, then along the horizontal line AB till we come to the graph at B. The line AB represents the number of centimetres in 4 inches, and reading its value on the centimetre scale we get 10·2

centimetres. Similarly, 6·2 inches gives us the line *CD*, which represents 15·7 centimetres.



The same graph enables us to change centimetres into inches. To find the value of 12 centimetres in inches, we

go along the centimetre scale till we come to 12 centimetres, then vertically upwards along the line *EF* till we come to the graph at *F*. *EF* represents the number of inches in 12 centimetres, and reading its value on the inch scale we get 4·7 inches. Similarly, 16·5 centimetres = 6·5 inches.

The graphs given on page 150 may be used to convert weights and measures from the British system to the metric system or *vice versâ*. The student may take 1 inch on the squared paper to represent 1 unit, 10 units, or 100 units, as suits his purpose, but he should remember that the results obtained in this way are only approximate, and that the larger the scale used the more accurate is the result.

Exercise 59.

(1) The population of a town in 1841 was 3900; in 1851, 4230; in 1861, 4540; in 1871, 4850; in 1881, 5150; in 1891, 5450; in 1901, 5740. Plot a curve showing these, and deduce approximately the population in 1856 and in 1876.

(2) For increases of temperature of 4, 8, 12, 16, 20, 24 degrees, a rod expands by ·55, 1·1, 1·65, 2·2, 2·75, 3·3 millimetres respectively. Plot a curve showing the connection between these values, and find the expansion for increases of 7, 10, and 22 degrees.

(3) Assuming that 10 kilograms = 22 lb., find graphically how many kilograms equal (1) 4·5 lb., (2) 17·6 lb.; and how many lb. equal (1) 6·7 kilos., (2) 12·4 kilos.

(4) Draw a graph showing the connection between the simple interest at 4 per cent. per annum on a given principal and the time; and find in what time the interest on £100 is £45, and what is the interest at the end of 7½ years.

(5) Find, by a graphical method, the number of pints equal to 7·4 litres and to 10·3 litres, and the number of litres equal to 8·1 pints and to 13·6 pints. (10 litres = 17·6 pints.)

(6) A cyclist goes at the rate of 12 miles an hour. Find, graphically, how far he goes in 3 hours 15 min., and how long he takes to go 63 miles.

(7) The areas of squares having sides of 1, 3, 5, 7, 9 inches are 1, 9, 25, 49, 81 square inches respectively. Draw a curve showing the change of area as the side varies, and derive the areas of squares having sides of 4, 6, and 7½ inches.

(8) If 3 francs per kilo. = 1s. 1d. per lb., draw a graph by means of which francs per kilo. can be converted into pence per lb., and find the equivalent in pence of 4 fr. 80 c. per kilo.

(9) If an article is sold for £5 above its cost price 3 °/₀ is gained. Draw a graph showing the connection between the gain per cent. and the total gain, and find from it at how much above cost price it would be necessary to sell the article to gain 7½ °/₀, and what would be the amount gained °/₀ by selling it at £9 above cost price.

(10) The distances passed over by a moving body in 2, 4, 6, 8, 10, and 12 seconds are 1, 4, 9, 16, 25, and 36 yd. respectively. Draw a curve illustrating this, and derive the space covered in 7, 3½, and 11 seconds.

(11) Find by squared paper the equivalent per litre in French money of 17s. 6d. per gallon, assuming that 12s. per gallon is equivalent to 3 fr. 30 c. per litre.

(12) The square roots of 1, 3, 5, 9, 12, and 16 are 1, 1·73, 2·24, 3, 3·46, and 4 respectively. Draw a curve representing these, and from it find the square root of 4, 10, and 13·5.

(13) Find, graphically, which is the cheaper: silk at 4 fr. 20 c. per metre or silk at 2s. 11d. per yard. (2 francs per metre = 1s. 5d. per yard.)

(14) A certain principal lent at simple interest amounts in 3 years to £230, and in 7 years to £270. Find, graphically, the principal, what it amounts to in 4½ years, and in how many years it will amount to £260.

(15) The speed of a moving body at the end of 1, 3, 5, 7, 12, and 16 seconds is 4½, 5½, 7½, 10, and 12 ft. per second respectively. Plot a curve showing its speed, and find its speed at the end of 5 and 13½ seconds, and after how many seconds will its speed be 7 and 11½ ft. per second.

(16) If an article be sold for five guineas 5 °/₀ is gained. Find, by a graphical method, (1) the cost price, (2) how much per cent. would be gained by selling it for £6, (3) at what price it should be sold to gain 25 per cent., and (4) at what price it should be sold to lose 15 per cent.

AVERAGES AND STATISTICS.

If I spent £5 one week and £3 the next week, we might say that, *taking one week with the other*, I had spent £4 per week.

The same idea would be expressed by the statement that I had spent £4 per week *upon the average.*

It is evident that we might find this *average* (£4) by adding together £5 and £3, and dividing by 2. Hence the method of obtaining the average of several numbers or quantities is to add them together and to divide by the number of numbers or quantities.

Referring again to our concrete example, it is evident that £5 is as much *greater* than £4 as £3 is *less* than £4. Similarly, if we find the average of 14, 7, 9, and 18—namely, 12—we see that the sum of the differences between 14 and 12, and 18 and 12, is equal to the sum of the differences between 7 and 12, and 9 and 12. Hence, we may say, the **average** or 'mean' of several numbers is an intermediate number, such that the sum of the excesses of the numbers above that average is equal to the sum of the defects of the numbers below it.

Examples in Averages: (1) *Find the average of* 80·68, 37·156, 9·371, *correct to three places of decimals.*

$$\begin{array}{r} 80{\cdot}68 \\ 37{\cdot}156 \\ 9{\cdot}371 \\ \hline 3)127{\cdot}207 \\ \hline 42{\cdot}402 \end{array}$$ Average.

(2) *Find the average of* 3·75, 0, $8\frac{2}{9}$, $7{\cdot}4\dot{3}\dot{8}$, *correct to three places.*

As the answer has to be correct to *three* decimal places, carry each item to *five* places, to ensure accuracy.

$$\begin{array}{r} 3{\cdot}75 \\ 0 \\ 8{\cdot}22222+ \\ 7{\cdot}43838+ \\ \hline 4)19{\cdot}41060 \\ \hline 4{\cdot}85265 \end{array}$$

$\therefore$ Average = 4·853 correct to three places.

Exercise 60.

Find the averages of the following numbers:

(1) (*a*) 79, 0, 134, 121, 96; (*b*) 192, 486, 700, 13, 241, 88.

(2) 756, 804, 915, 1000, 648, 575, 480, 521, 836, 950.

(3) The natural numbers from 60 up to 90 inclusive.

(4) The natural numbers from 100 down to 50 inclusive.

[Is there any easy way of obtaining the answers to the last two questions?]

(5) (*a*) $2\frac{1}{2}$, $5\frac{1}{4}$, $7\frac{3}{8}$, 4, $3\frac{9}{8}$; (*b*) $152\frac{5}{8}$, $71\frac{1}{4}$, $\frac{17}{18}$, 100, $243\frac{7}{12}$.

(6) (*a*) $3004\frac{7}{11}$, $568\frac{3}{8}$, $\frac{19}{118}$, 26, $61\frac{1}{32}$; (*b*) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{11}{12}$.

Find the averages of the following, correct to two decimal places:

(7) 78·65, 4·847, 19·75, 48·2, 23·016, 35·427.

(8) 6·593, 7·248, 8·916, 9·841, 10·016, 12·372.

(9) ·1, ·2, ·3, ·4, ·5, ·6,..... up to 1 inclusive.

(10) A man travelled 30 miles in one day, 36 the next, 40 the third, 28 the fourth, 30 the fifth, and 16 the sixth. What was the average number of miles he travelled per day?

(11) A gentleman's income for 1893 was £300; for 1894, £560; for 1895, £480, 10s.; and for 1896, £390, 16s. 8d. Find his average yearly income.

(12) A shopman's takings for the first five days of the week are £5, 7s. 3d., £4, 18s. 7d., £6, 11s. 9½d., £6, 5s. 7½d., and £6, 16s. 5d. respectively. How much must he take on Saturday to make the daily average for the six days £7?

(13) A traveller is allowed £1 per day for his expenses. At the end of July he finds that he is 3s. 10½d. out of pocket for that month. Find his average daily expenditure for that month.

(14) A traveller's railway expenses for 5 days were 3s. 6d., 2s. 3d., 8s. 9d., 3s. 4d., and 5s. 6d. respectively, and his hotel bills were 12s. 6d., 10s. 6d., 13s., 11s. 6d., and 13s. 6d. Find his average daily expenditure.

(15) A man's weekly wage is 43s.; the keep of his family of seven costs him 3s. 9½d. per head per week; and in 27 weeks he has saved £3, 1s. 10½d. What is the average weekly amount of his other expenses?

(16) Find correct to one decimal place of pence and of miles the entries required for the blank spaces in the following extract from a railway company's report:

	No. of Passengers.	Aggregate Miles Carried.	Fares Earned.	Average Fare per Passenger.	Average Distance Travelled.
			£		
1st class	26912	2151519	22762	...s. ...d.	.....miles.
2nd class	64827	3319671	26478	...s. ...d.	.....miles.
3rd class	294838	10760739	50790	...s. ...d.	.....miles.

(17) The following table gives the wages bill for the year, and the average weekly number of employees, in each of the textile industries of the United Kingdom for the year 1906. Find the average annual earnings per head in each case, to the nearest half-sovereign.

Industry.	Average Weekly Number Employed.	Aggregate Wages Bill for the Year.	Annual Earnings per Head.
		£	£ s.
Cotton	212503	10195229	
Woollen and Worsted .	110554	4418385	
Linen	45699	1343835	
Jute	14060	487911	
Silk	13920	481536	
Hosiery	19555	755323	
Lace	8025	435736	
Carpet	11067	450914	
Hemp	8379	300835	
Smallwares . . .	5634	166630	
Flock and Shoddy . .	2226	103280	
Elastic Web . . .	1313	48629	
Hair	880	27027	
Other Textile . . .	4289	167291	
Fustian and Cord Cutting	838	25382	
Bleaching, Printing, &c. .	55925	3060492	
All Textile Industries .			

(18) In the following table calculate the averages required in the blank columns and at the foot of each column. Find the answers, correct to the nearest farthing.

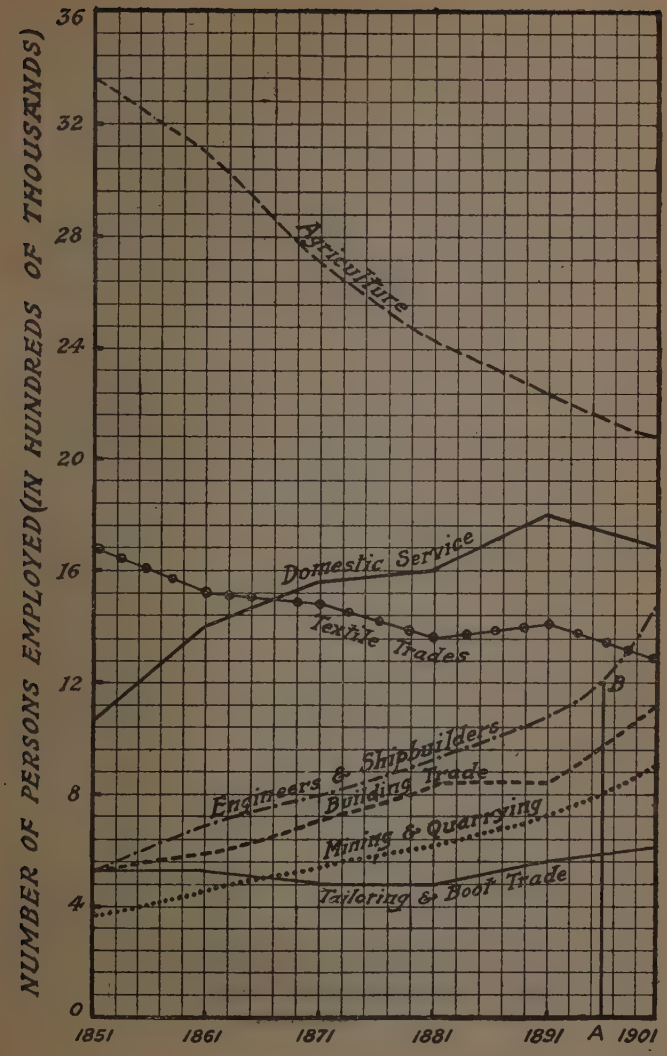
EDUCATION GRANTS FOR CERTAIN SCHOOLS.

	Total Grants.						Average Attendance.		Grant per Head on Average Attendance.	
	1910.			1909.			1910.	1909.	1910.	1909.
	£	s.	d.	£	s.	d.				
(1)	270	6	6	286	2	0	233	242		
(2)	290	5	0	298	0	6	244	245		
(3)	848	3	10	866	19	11	670	683		
(4)	149	8	0	166	16	0	128	138		
(5)	355	11	0	298	4	0	304	254		
(6)	693	2	2	626	4	2	506	487		
(7)	477	18	6	478	0	6	407	409		
(8)	245	1	5	240	0	7	188	190		
(9)	754	4	5	733	6	5	632	598		
(10)	279	13	6	268	18	6	239	233		
(11)	198	5	6	202	10	0	163	164		
(12)	583	11	0	552	10	0	500	476		
(13)	1307	10	10	1274	4	8	986	985		
Totals.										

Numerous tables of statistics for practice will be found in the *Daily News* and *Daily Mail* Year Books.

GRAPHIC REPRESENTATION OF STATISTICS.

When a quantity varies from day to day, or from year to year, we can make a figure to show this variation with the aid of a sheet of squared paper. In the accompanying figure we have shown the variation in the number of persons employed in certain trades in the United Kingdom from 1851 to 1901 as given in the census returns. Such a figure is called a graph. It shows at a glance the increase or decrease in the


NUMBER OF PERSONS EMPLOYED (IN HUNDREDS OF THOUSANDS)
36
32
28
24
20
16
12
8
4
0
Agriculture
Domestic Service
Textile Trades
Engineers & Shipbuilders
Building Trade
Mining & Quarrying
Tailoring & Boot Trade
B
A
1851
1861
1871
1881
1891
1901

number of people engaged in any of the trades selected. We can also use the graph to tell the approximate number of persons engaged in any of the given trades in any year from 1851 to 1901. For example, the approximate number of persons engaged in engineering and shipbuilding in 1896 was 1,200,000. This is obtained by going along the horizontal time line to 1896, then vertically till we come to the graph marked Engineers and Shipbuilders. The vertical line AB in the figure represents the number employed, and its value is found on the vertical scale.

THE AVERAGE RATE OF INCOME-TAX.

The financial year of the British Government ends on 5th April. When a change is made in the income-tax, as in 1910, when the rate on unearned incomes was increased from 1s. to 1s. 2d. per £, accountants, factors, and others find it necessary to calculate the average rate for the six months, 11th November to 15th May, in order to deduct the income-tax from ground-rents, and interest of all kinds, which run from Martinmas to Whitsunday. The calculation should be done thus:

From 11th November to 5th April gives us 145 days at 1s.
= 145s.
" 5th April to 15th May gives us 40 days at 1s. 2d.
= 46·6s.
Total for 185 days
= 191·6s.

191·6s. ÷ 185 = 1s. 0·42810d. = average rate of income-tax.

EXAMPLE.—*Calculate the income-tax to be deducted from ground-rents, amounting to* £100, *payable for* 6 *months ending* 15*th May* 1910.

Tax to be deducted = £100 × 1s. 0·42810d.
= £5, 3s. 7d.

Exercise 61.

(1) A cricketer in successive innings scored 21, 35, 2, 76, 0, 146, 63, 10, 29, 8, 98, 52. Find his average score per innings.

(2) Find the average daily expenses of a man who spends £400 a year.

(3) The forenoon attendances at a certain school for a week were 45, 43, 47, 46, 46. The afternoon attendances for the same week were 42, 41, 46, 47, 43. Find the average attendance (1) in the forenoon, (2) in the afternoon, (3) for the week.

(4) Of 13 boys leaving school for business, 6 were of the age of 14 years, 5 of 15, and 2 of 16. What was the average age?

(5) Of 21 men, 13 lose £116, 7s. 8d. each, and 8 lose £93, 0s. 9d. each. What is the average loss per man?

(6) A man buys 25 sheep for £36, and 30 more for £46. What will he gain or lose on each sheep if he sells them at £1, 10s. 6d. each?

(7) A man buys 22 shares at £6 each, and 78 at 24s. each. What does he lose per share by selling them all at 34s. each?

(8) The average passenger fare on a railway is 1½d. a mile, the average journey is 12 miles, and the average weekly revenue is £710,306. Find the average number of passengers in a week.

(9) The average of 7 results is 16; the average of the first two is 18, and of the last two 12. What is the average of the rest?

(10) A man's average daily expenditure from Monday to Friday inclusive is £2, 1s. 6d.; he spends £1, 15s. 6d. on Monday, and £2, 5s. 6d. on Saturday. What is his average daily expenditure from Tuesday to Saturday inclusive?

(11) The expenses of two parliamentary candidates, R and S, were: R, £1307, 18s. 9d., giving an average of 5s. 3¾d. per vote; S, £1516, 10s., giving an average of 5s. 7½d. per vote. Which candidate was elected, and by what majority?

(12) If a boy has made 196 runs in 13 innings, how many must he make in the next two innings so as to have an average of 19?

(13) Three schools educate 75, 33, and 31 children at an annual average cost of £2, 10s., £3, 10s., £3, 15s. respectively. They are united, and an annual saving is thereby effected of £141, 5s. Find the annual average cost after their union.

(14) A train travels at the rate of 40 miles an hour for 10 minutes, and then at 45 miles an hour for 15 minutes. What is the average per hour for the whole time?

(15) Two steamers leave a place at the same time for the same port, distant 1260 miles. The faster steamer averages 11½ miles an hour, and arrives at the port 28 hours before the other. Find the average speed of the slower steamer.

(16) A tourist set out with £19, 10s. At the end of six days he found that the money he had spent was just equal to ⅔ of what remained. What was his average daily outlay?

(17) A gentleman's income for the first five months of the year was £306 per month; for the next four months, £280 per month; and for the rest of the year, £250 per month. Determine his average monthly income.

(18) At an entertainment given on three following evenings the receipts were as follows: On the first evening, £80; on the second, 40 per cent. less than this; and on the third evening, one-fourth more than on the second evening. What were the average receipts per evening?

(19) A man earns 4s. 6d. on Monday, ⅓ more than this on Tuesday, 5s. 8d. on Wednesday, on Thursday 25 per cent. more than on Wednesday, 6s. on Friday, and the average for Monday and Tuesday on Saturday. What are his average daily earnings?

(20) A company made a profit of £4000 in August, in September 12½ per cent. more, and in October 25 per cent. more than the average for August and September. Determine the average profits for the three months.

(21) A man earns 6d. per hour for overtime work. In one week he works 30 hours overtime, in the second 25, in the third 12½, and in the fourth 24. What are his average weekly overtime earnings?

(22) A commission agent received 2½ per cent. on all sums collected by him for his employers. In January he received for his firm £800, in February £600, in March £560, in April £600, in May £700, and in June £840. What was his average monthly income for the half-year?

It is important to notice that the average of a set of averages need not be the correct average of the whole of the original numbers.

The average of two numbers is 8, *and the average of three others is* 6. *Find the average of the five numbers* (*or the gross average*).

Since the average of the first two numbers is 8, the sum of the two numbers must be $8 \times 2 = 16$; and since the average of the other three numbers is 6, the sum of the three numbers must be $6 \times 3 = 18$. Therefore, the sum of the five numbers must be $16 + 18 = 34$, and the average of the five numbers must be $34 \div 5 = 6\frac{4}{5}$.

But if we take the average of the two averages given we get $\frac{1}{2}$ of $(8 + 6) = \frac{1}{2}$ of $14 = 7$, which is incorrect.

Although it is often wrong to take the average of a set of averages for the gross average, yet in certain cases where the saving of labour is great and the error committed is small such is the method adopted. In meteorological returns, for instance, the average height of the barometer for a day in any month is obtained in the ordinary way, by adding up the inches for every day in the month and dividing the sum by the number of days. But in striking the yearly average it is usual to add up the averages for the several months and to divide the sum by 12. This, of course, would be an absolutely correct method if all the months had the same number of days. But this is not so, and, therefore, the result obtained is not quite correct. The error committed, however, is exceedingly small.

EASY STOCKS AND SHARES.

The money invested in a business is called the **capital**, and may be provided by one person, or by a number of shareholders, who form a company.

Suppose a company were formed with a capital of £50,000, divided into 5000 shares of £10 each. One man might hold ten of these shares, another fifty, and so on; and the profits of the business would be divided among the shareholders in proportion to the number of shares they held, so that each shareholder would receive as profit a certain percentage on

each £10 share he possessed. If the profits were 4 per cent., for example, he would receive 8s. on each £10 share. (It might be that the whole of the £10 per share would not be required from the shareholders, but only some portion, say £7. In that case the shares would be said to be not fully paid up; and if the company paid a dividend of, say, 5 per cent., the 5 per cent. would be calculated on the amount paid up—that is, on the £7, *not* on the £10.)

If a shareholder wished to withdraw from the business and have his money again, he would have to find some one willing to buy his shares. The price he would obtain for them would probably not be exactly the same as he gave for them. If the business were not very prosperous, he would have to sell them for less than their nominal value; but if it were succeeding well, he would probably obtain more.

When shares can be bought for *less* than their nominal value, they are said to be **at a discount** or **below par**; when *more* than their nominal value has to be paid for them, they are said to be **at a premium** or **above par**; when they fetch *exactly* their nominal value, they are said to be **at par.**

Shares are generally bought and sold through a broker, who charges a certain amount of **brokerage** from both parties; so that the seller obtains rather less, and the buyer pays rather more, than the market price for his shares.

As a rule, shares are bought and sold entire; that is, fractions of shares are not obtainable. A company may, however, if its shares are fully paid up, convert them into **stock,** and thus allow any quantity of stock to be bought or sold. The price of stock is generally quoted at so much per cent. Thus, if Caledonian Railway stock be quoted at 153, this means that £100 of stock costs £153 of money.

Governments frequently borrow money during times of unusual expenditure, and issue stock, on which interest at a certain rate per cent. is promised, as security to the lenders. Like shares, this Government stock is frequently

bought and sold. The money which has been borrowed at various times by the British Government constitutes our **National Debt**; it is also called **the Funds**, or **Consols.**

Ex. 1. *How much stock can a man purchase by investing* £2520 *in the* $2\frac{3}{4}$ *per cents. at* 105?

£105 purchases £100 stock; therefore, £2520 will purchase—

$$£100 \times \frac{2520}{105} = £2400 \text{ stock.}$$

Notes.—(1) It will be observed that the item '$2\frac{3}{4}$ per cent.' does not enter into this question at all; it merely serves to describe the nature of the stock.

(2) If brokerage at $\frac{1}{8}$ per cent. were to be charged, the buyer would have to pay £$105\frac{1}{8}$ for £100 stock, and the amount would be—

$$£100 \times \frac{2520}{105\frac{1}{8}} = £2397 \text{ stock (approx.).}$$

Ex. 2. *If I hold* £650 *stock in the* 3 *per cents., what interest do I receive?*

For £100 stock I receive £3 interest; hence, for £650 stock I receive—

$$£3 \times \frac{650}{100} = £19, \text{ 10s.}$$

Ex. 3. *If I invest* £1344 *in* $2\frac{1}{4}$ *per cent. stock at* 96, *what income shall I derive?*

Here we may find (1) the amount of stock bought, using the method employed in Ex. 1; and then (2) the income derived from that stock, using the method employed in Ex. 2.

Or, we may find the required income directly. £96, when invested, produces £$2\frac{1}{4}$ interest; therefore, the interest on £1344 equals—

$$£\frac{9}{4} \times \frac{1344}{96} = £31, \text{ 10s.}$$

Ex. 4. *What money shall I receive for* £2600 *stock in the* $3\frac{1}{2}$ *per cents. at* $115\frac{7}{8}$, *if* $\frac{1}{8}$ *per cent. brokerage is charged?*

For £100 stock I really receive £$115\frac{3}{4}$; therefore, for £2600 stock I receive—

$$£\frac{463}{4} \times \frac{2600}{100} = £3009, \text{ 10s.}$$

Ex. 5. *What must I pay for* £2840 *stock in the* $2\frac{1}{2}$ *per cent. Consols at* $94\frac{1}{4}$ (*brokerage* $\frac{1}{8}$)?

For £100 stock I really pay £$94\frac{3}{8}$; therefore, for £2840 I pay—

$$£\frac{755}{8} \times \frac{2840}{100} = £2680, 5s.$$

Ex. 6. *What sum of money must be invested in India* 3 *per cents. at* £$106\frac{1}{8}$ *to secure an income of* £120 (*brokerage* $\frac{1}{4}$ *per cent.*)?

To secure an income of £3, £$106\frac{3}{8}$ must be invested; therefore, to secure an income of £120 the amount invested must be—

$$£\frac{851}{8} \times \frac{120}{3} = £4255.$$

Ex. 7. *Which is the better investment—the* $2\frac{3}{4}$ *per cents. at* 108, *or the* $2\frac{1}{2}$ *per cents. at* 104?

This may be worked in several ways.

(1) We know that in the former stock £108 produces £$2\frac{3}{4}$, and in the latter £104 produces £$2\frac{1}{2}$. Hence, we can find what £1 would produce in each case, and compare the results.

(2) We can find what £100 would produce in each stock, and compare the results.

(3) Knowing that £108 produces £$2\frac{3}{4}$ in the former stock, we might find what £104 would produce in that stock. Having ascertained this, and knowing that £104 produces £$2\frac{1}{2}$ in the latter stock, we see at once which is the better investment. *Or*—

(4) We might find what £108 would produce in the $2\frac{1}{2}$ per cents., and at once make the comparison.

(5) We might find how much must be invested in each stock to produce £1 interest, and compare the results. The one in which the *less* amount has to be invested is, of course, the better investment.

The methods (3), (4), and (5) involve, as a rule, less working than (1) and (2).

Ex. 8. *A man buys* £8000 *five per cent. railway stock at* 175. *What return per cent. does he obtain on his outlay?*

£175 brings in £5; therefore, £100 brings in—

$$£5 \times \frac{100}{175} = 2\frac{6}{7} \text{ per cent.}$$

Note.—The amount of stock does not enter into the question.

Ex. 9. *A man invested* £1196 *in Consols at* 104, *and sold out at* 110. *What profit did he make?*

On every £104 invested he made a profit of £6; therefore, the profit on an investment of £1196 equals—

$$£6 \times \frac{1196}{104} = £69.$$

Exercise 62.

How much stock is obtained by investing

(1) £2520 in the $3\frac{1}{2}$ per cents. at 105?
(2) £2275 in the 4 per cents. at $113\frac{3}{4}$?
(3) £270, 8s. in the $2\frac{1}{4}$ per cents. at $15\frac{1}{2}$ discount?
(4) £2523, 10s. in the $3\frac{3}{4}$ per cents. at a premium of 3?
(5) £1796, 12s. in the $2\frac{1}{2}$ per cents. at $86\frac{1}{4}$ (brokerage $\frac{1}{8}$)?

What annual dividend will be received from the following stock:

(6) £3000 of $3\frac{1}{2}$ % stock?
(7) £4150 of $2\frac{3}{4}$ % stock?
(8) £4125 of $3\frac{3}{4}$ % stock?
(9) £1612, 10s. of $2\frac{1}{2}$ % stock?
(10) £5406, 5s. of $4\frac{1}{2}$ % stock?
(11) £650 of $3\frac{3}{4}$ % stock?

What annual dividend shall I receive if I invest

(12) £2880 in the $3\frac{1}{4}$ per cents. at 96?
(13) £277, 15s. in the $3\frac{1}{2}$ per cents. at 101?
(14) £630 in the $2\frac{3}{4}$ per cents. at $94\frac{1}{2}$?
(15) £656, 5s. in the $4\frac{1}{4}$ per cents. at 105?
(16) £4410 in the $2\frac{1}{2}$ per cents. at $94\frac{3}{8}$ (brokerage $\frac{1}{8}$)?
(17) £4255 in the 3 per cents. at $106\frac{1}{4}$ (brokerage $\frac{1}{8}$)?

What shall I pay for

(18) £7250 of $2\frac{3}{4}$ % stock at 89?
(19) £6375 of 4 % stock at 104?
(20) £732, 10s. of $3\frac{3}{4}$ % stock at $95\frac{1}{2}$?
(21) £3206, 5s. of $3\frac{1}{2}$ % stock at $88\frac{1}{2}$?
(22) £487, 10s. of $3\frac{1}{2}$ % stock at $90\frac{5}{8}$ (brokerage $\frac{1}{4}$)?
(23) £1023, 13s. 4d. of 4 % stock at $108\frac{1}{4}$ (brokerage $\frac{1}{8}$)?

What must I give for

(24) £3500 of $2\frac{3}{4}$ % stock at 96?
(25) £4500 of 3 % stock at $93\frac{1}{4}$?
(26) £650 of 4 % stock at $115\frac{3}{8}$?
(27) £963, 4s. of $2\frac{1}{2}$ % stock at $93\frac{3}{4}$?
(28) £698, 10s. of $2\frac{1}{4}$ % stock at $89\frac{7}{8}$ (brokerage $\frac{1}{4}$)?
(29) £1000 of 3 % stock at $95\frac{1}{4}$ (brokerage $\frac{1}{4}$)?

What sum must I invest to obtain an annual income of

(30) £120 from $2\frac{1}{2}$ % stock at 80?
(31) £1056 from $5\frac{1}{2}$ % stock at 126?
(32) £105, 2s. from 4 % stock at $101\frac{1}{2}$?

(33) £60 from 3 °/₀ stock at 91⅝?
(34) £268, 2s. from 3½ °/₀ stock at 92⅜ (brokerage ⅛)?
(35) £501, 10s. from 4¼ °/₀ stock at 108⅝ (brokerage ⅛)?

Which is the better investment:

(36) 3 °/₀ stock at 80, or 3½ °/₀ stock at 91?
(37) 3 °/₀ stock at 64½, or 5 °/₀ stock at 102¾?
(38) 4 °/₀ stock at 120, or 2½ °/₀ stock at 75?
(39) 1¼ °/₀ stock at 51, or 4$\frac{7}{16}$ °/₀ stock at 154?

What interest per cent. do I get if I invest

(40) In 3 °/₀ stock at 80?
(41) In 5 °/₀ stock at 110?
(42) In 3½ °/₀ stock at 94½?
(43) In 2½ °/₀ stock at 82¼?

(44) Find the income obtained by investing £18,669 in 3 per cent. stock at 95⅛ (brokerage ⅛ per cent.).
(45) A person invests £1271 in 3 per cent. stock at 93. How much will he gain by selling out at 94⅛?
(46) A person invests £1000 in stock which sells at 92½. At what price must he sell out to clear £50?
(47) Investing a sum of money in a certain 3 per cent. stock would bring the same income as lending it at 4 per cent. What is the stock selling at?
(48) Certain £5 shares are selling at £6, 12s. per share. How many can I buy for £318 (brokerage = 6d. per share)?
(49) A company pays a dividend of 8 per cent., and its £5 shares are selling for £12. What percentage will an investor get for his money?
(50) If in 5 years I receive £560 as dividends on stock in the 3½ per cents., what amount of stock do I hold?
(51) What income shall I derive from £3600 if I invest ⅔ of it in the 3½ per cents. at 80, and the remainder in the 4½ per cents. at 90?
(52) I invest £1440 in 3½ per cent. stock, and receive a dividend of £52, 10s. Find the price of the stock.
(53) What is the price of 2½ per cent. stock which brings me in 2⅝ per cent. on my investment?
(54) What is the difference between the yearly income produced by investing £215 in the 2½ per cents. at 86, and in the 3¼ per cents. at 129?
(55) What is the annual income derived from investing £6050 in the 3 per cents. at 90⅝ (brokerage ⅛), after deducting income-tax at 1s. 2d. in the £?

EXAMINATION PAPERS.

The following questions in Commercial Arithmetic have been selected from the Leaving Certificate Examination papers. The first ten sets should be done mentally, half-an-hour being allowed for each set.

I.

(1) *Add these across.*

7924	3816	5204	768	1398	4265	5600	832
3872	1405	6234	1733	4265	836	1274	3508
138	5672	3088	5671	8824	5903	7800	1833
5243	739	1527	8933	1076	2694	330	4165
2756	4193	574	2707	385	7707	8042	5247

(2) *Find the value of:*

	£	s.	d.
5 per cent. of	104	7	6
2½ " "	164	10	0
1 " "	20	8	4
12½ " "	431	5	4
7 " "	650	0	0
12 articles at	0	1	6½ each
15 " "	10	4	0 "
20 " "	2	7	9 "
24 " "	0	0	6¾ "
50 " "	0	2	6 "
88 " "	0	0	0¾ "
100 " "	1	7	6 "
144 " "	0	0	2¼ "

(3) *Add:*

£	s.	d.
1437	14	3
960	11	6
4425	8	10
736	15	6½
2034	10	8
6715	9	11
8044	16	10
736	10	4¾
1935	19	9
5027	3	10½
293	17	4
7164	14	3

II.

(1) *Add:*	(2) *Write down the values of:*
16459	5 per cent. of £364, 10s.
3947	2½ " £198, 3s. 4d.
70584	15 " £47.
679	4 " £735.
31886	1 dozen articles at 6¾d. each.
49375	2 " " 1s. 1½d. each.
8705	1 gross articles at 4½d. each.
66993	25 articles at 4s. 6d. each.
24388	150 " 2s. 6d. each.
71690	240 " 1s. 2½d. each.
178	$\frac{3}{4}+\frac{5}{8}-\frac{7}{12}$.
62949	·05 − ·00005.
20796	·05 × ·00005.
384	·05 ÷ ·00005.
16097	·164 × 125.
44968	·164 ÷ 125.
1475	Express £4·725 in £, s. d.
39587	" £16·1875 "
641	" 18s. 6d. as a decimal of £1.
83649	" 4s. 3d. as a vulgar fraction of £1.
30076	" 66 yd. " " 1 mile.
8364	" 1 qr. 12 lb. " " 1 ton.
92873	" 3·5 metres + 6·8 decimetres in centimetres.
15946	" 4 kilograms + 47 grams in kilograms.

III.

(1) *Add the following sums vertically and horizontally and find the grand total:*

£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
207	14	3	321	15	10	28	17	9	631	5	7	520	0	6	500	15	6
512	9	9	719	4	7	371	7	3	72	16	8	413	12	5	142	3	4
176	10	4½	445	13	8	414	13	6¾	114	14	11	240	5	7½	97	10	3½
527	17	6	873	9	2	801	5	4	138	3	2	17	18	9	370	18	9
389	7	9¼	96	19	8½	990	18	0½	501	0	9	125	3	4¾	412	17	7½

(2) *Write down the values of:*

A dozen articles at 5s. 8¼d. each.
One article at £4, 7s. 9d. per doz.
47165302 – 17854667.
£10417, 15s. 3½d. – £9923, 4s. 10¼d.
571275 ÷ 25.
4389 × 125.

$\frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\frac{7}{16}$.
3·725 + ·0065 + 41·85.
Express £1·775 in £, s. d.
" £3·60625.
" 15s. 9d. as a decimal of £1.

IV.

(1) *Add the following sums in rows and in columns and find the grand total:*

£	s.	d.	£	s.	d.	£	s.	d.
4325	16	3½	376	11	4	1721	14	3¾
1743	9	10	2194	17	6	836	5	7
425	17	11¼	873	4	9¾	2571	13	4½
3906	10	8¾	6214	9	6½	1936	15	8¼
4778	18	7	938	14	10¼	399	19	11¾
837	13	4½	1724	12	6	5293	12	6¼
1275	7	9	4286	8	11½	85	3	5
3321	6	4	734	2	2¼	537	18	10

(2) *Write down the values of:*

365 times 4d.
47963 × 101.
78314 × 125.
5 tons 13 cwt. 1 qr. 14 lb. ÷ 7.
7 furlongs 8 poles 3 yd. 2 ft. ÷ 12.
4 per cent. of £3765.
15 per cent. of £1446.

(3) *Express as decimal of* £1:

18s. 6d.
£5, 7s. 4½d.
£2, 13s. 8½d.

(4) *Express in* £, s. d. *to the nearest farthing:*

£1·995.
£7·879.

V.

(1) *Write down the values of:*

8¾ per cent. of £2500.
$3\frac{5}{16}$ per cent. of £7000.
4 per cent. of £76, 5s.
144 times 6¾d.
365 times 2s. 4d.
·0048 × ·0125.
4·78365 – ·08366.
51629 × 55.

(2) *Express as a decimal of* £1, *corrected to the nearest figure, in the third decimal place:*

£4, 17s. 8½d.
£1, 9s. 11½d.
£3, 14s. 7d.

(3) *Express in* £ s. d. *to the nearest farthing:*

£2·716. £4·325.

(4) *Add the following columns:*

£	s.	d.	Yd.	Ft.	In.	Tons.	Cwt.	Qr.	Lb.	Oz.
427	3	10	31	2	11	14	10	1	11	6
78	11	6½	16	1	4	5	3	3	4	13
813	10	11	9	2	10	2	14	2	9	12
174	9	8¼	23	1	5	7	18	1	25	4
365	14	11	10	1	9	3	9	2	16	14
26	19	11¾	7	2	4	5	14	3	20	6
856	7	3	16	1	8	16	11	2	13	12
247	15	7½	8	0	6	5	3	0	7	10
139	8	4¼	14	2	2	2	10	2	14	5

VI.

(1) *Add the following sums of money, and subtract nine times the first line from your result:*

£	s.	d.
176935	19	11½
89277	17	6¾
618248	9	10½
975649	17	6
419365	13	4½
987349	9	8
126534	18	10¼
969417	11	11
876994	19	8¼

(2) *Write down the values of:*

234716953 × 11.
417296534 × 25.
643917258 × 12.
2716 × 34.
654678228 ÷ 12.
321936575 ÷ 25.
47149625 ÷ 125.

(3) *Calculate the following percentages:*

5 % of £416932.
4 „ £391165.
2½ „ £624163.
12 „ £6735.
95 „ £14500.

(4) *Write down the cost of:*

784 articles at 13s. 4d. each.
6 tons 16 cwt. 2 qr. at £12, 10s. per ton.
260 yards at 2d. per inch.

(5) *Express:*

456 centimetres + 32 decimetres in millimetres.
137 square centimetres as a decimal fraction of a square metre.
472 cubic metres as cubic millimetres.

(6) *What is the remainder when the following number is divided by* 9? 3721845023465.

VII.

(1) *Add the following sums, and express your answers in* £ s. d., *to the nearest farthing:*

£	£
476502·714	729641·824
158236·482	526771·347
70984·370	108995·732
829564·078	92684·503
291663·845	835916·477
179258·613	106941·385
3617·008	86435·789
450928·175	478160·158
69178·382	397307·512
	71011·977
	829356·455

(2) *How many*

Pence in £5, 10s. 6d.?
Inches in 8 yd. 2 ft. 4 in.?
Sq. poles in 2 acres, 3 roods, 25 poles?
Lb. avoir. in 3 cwt. 2 qr. 10 lb?

(3) *Write down the values of:*

(*a*) $\frac{8}{9} - \frac{8}{9} + \frac{7}{10}$; (*b*) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$; (*c*) $\frac{5}{12} \times \frac{8}{20} \times \frac{24}{25} \times$.
(*d*) 94·6 − 8·76554; (*e*) 2·5 × 3·2 × 6·3.

(4) *Find the prices of:*

3 dozen articles at 1s. $2\frac{1}{2}$d. each.
4 score " 5s. $1\frac{1}{2}$d. "
2 gross " $2\frac{3}{4}$d. "
240 " 1s. $6\frac{3}{4}$d. "

(5) How many square feet in a rectangle 3 ft. 6 in. by 2 ft. 6 in.?
Express as a decimal fraction of a square metre the area of a rectangle 43 cm. by 125 mm.
How many cubic millimetres in a cubic decimetre?

VIII.

(1) *Add:*

£	s.	d.
476	13	$10\frac{1}{2}$
9025	18	$4\frac{1}{4}$
708	6	$11\frac{1}{2}$
870	10	10
1795	15	9
2863	17	$8\frac{3}{4}$
9	14	3
4085	19	$4\frac{1}{4}$
16420	10	0
297	7	11
4263	0	$6\frac{1}{2}$

(2) *Add:*

Ac.	Roods.	Poles.
112	3	27
47	1	17
19	2	35
47	3	0
285	2	19
372	0	26
29	2	30
64	1	15
508	3	28
10	2	5
61	3	39

(3) *Add:*

$$34781 + 129625 + 4936 + 72091 + 60485 + 27987 + 3895.$$

(4) *Write down the cost of:*

87 articles at 2s. 6d. each.
45 " £1, 8s. "
256 " $1\frac{1}{2}$d. "
365 " $4\frac{1}{2}$d. "

(5) *Calculate these percentages:*

$2\frac{1}{2}$ per cent. of £6916, 10s.
$7\frac{1}{2}$ " £80, 8s. 4d.
25 " £468, 16s. 10d.
40 " £674582.

(6) *Write down the values of:*

$1270005 \times 99.$
$2916329 \div 13.$
$16 \times 110 \times 25 \times 30.$
$\frac{5}{6} - \frac{7}{12} + \frac{7}{8} - \frac{3}{4} + \frac{1}{24}.$
$36{\cdot}79 + 25{\cdot}816 - 9{\cdot}7245.$

IX.

(1) *Write down in another column, in pounds, shillings, and pence, the equivalents, to the nearest penny, of the sums in the given column, and add both columns:*

£
4·739
3·280
8·175
5·741
4·503
7·512

(2) *Write down the values of*

$62345 \times 71.$
$81436 \times 42.$
$71445 \div 15.$

(3) *Calculate the following percentages:*

6 °/₀ of £1395.
125 °/₀ of £2468.

(4) *Write down the costs of:*

46 articles at £1, 2s. 6d. each.
144 " 3¾d. "
76 " 1s. 4d. "

(5) *Find the values of:*

(*a*) 4·72 + 3·645 + 9·0641 ; (*b*) 127·416 − 39·2458 ; (*c*) 12·35 × 1·25.

X.

1. *Write down in another column, as decimals of* £1, *correct to three places, the sums given in this column, and add both columns:*

s.	*d.*
10	0
2	6
17	6
6	8
13	4
1	6
3	4½
7	9¾
4	2½
5	11

2. Add 72·36 + ·521 + 5416·2 + 964·2873 =

3. (1) If a dozen articles cost 15s. 6d., one article will cost?
(2) If a score of articles cost £3, 14s. 7d., one article will cost?
(3) If a gross of articles cost £2, 5s., one article will cost?
(4) If a hundred articles cost £13, 17s. 1d., one article will cost?

4. ⅜ of an estate cost £7434. Find cost of whole.

5. (1) 17 yd. 2 ft. 5 in. × 12.
(2) 34 acres, 2 roods, 20 poles ÷ 5.
(3) 8 cwt. 2 qr. 17 lb. *less* 2 cwt. 3 qr. 23 lb. 8 oz.

6. (1) 4178 × 6s. 8d.
(2) 3½ °/₀ of £250000.
(3) 3⅓ ft. × 4½ ft.
(4) 706·452 − 28·46271.

XI.

(1) Find the cost of 126,516 articles at £2, 3s. 6d. per dozen.

(2) *Find the total of the following bill:*

		£	s.	d.	
35	articles at	1	6	3	each.
15½	"	0	11	3½	"
6¾	"	0	17	6	"
4⅝	"	3	15	7½	"

(3) A tradesman by selling an article for 6s. 9d. gains 35 per cent. What would he have gained per cent. if he had sold it for 8s. 3d.?

(4) Find the simple interest on £5006, 13s. 4d. for 2¾ years at 3¼ per cent.

(5) A path, 4½ feet wide, is made round a lawn 12 yards long and five yards wide. How many square yards are there in the surface of the path?

(6) If eggs are bought at 7d. per dozen, how many must be sold for a shilling to make, as nearly as possible, a profit of 20 per cent.?

(7) The weight of a freshly coined sovereign is 123·27 grains, but when it is reduced to 122·5 grains it is returned to the Mint to be recoined. If 100,000 such sovereigns are returned to the Mint, find the loss of value, measured in standard sovereigns, correct to 3 decimal places.

(8) A map is on the scale of 6 inches to a lineal mile. How many acres does a square inch represent?

(9) A dealer buys Ceylon tea at £12 per cwt., and Chinese tea at £9, 6s. per cwt., mixing them in the proportion of 3 of the former to 7 of the latter. Find, to two decimal places, how much per cent. he will gain or lose by selling the mixture at 2s. per lb.

(10) Assuming that a metre contains 39·37 inches and that £1 is worth 25·2 francs, find, correct to three decimal places of 1d., the price per yard which corresponds to a price of 5 francs per metre.

Hence find the value of 500 yards of lace worth 20 francs per metre.

XII.

(1) Find (1) the cost of 27938 things at £4, 12s. 9¾d. each; and (2) the rent of 47 acres 3 roods 8 poles at £5, 6s. 3d. an acre. (Answers obtained by practice will be preferred.)

(2) Find the sum which, along with simple interest at four per cent. per annum, will amount to £92, 16s. in four years.

(3) In a school of 1200 the number presented for an examination was 20 per cent. of the whole, while the numbers passing in the first and second classes were respectively $12\frac{1}{2}$ and $32\frac{1}{2}$ per cent. of those presented. Find the number presented and the number passing in each class.

(4) An oblong piece of land is 165 yards long and 143 yards broad. Find (1) its area in acres, roods, and poles; (2) its rent at £3, 6s. per acre.

(5) The area of Scotland is 29785 square miles, and the average number of inhabitants in each square mile is 139. Find the whole population.

(6) A merchant receives a consignment of 140 bags of coffee, the gross weight being 10 tons 4 cwt., and the tare (i.e. the weight of the bag, &c.) $2\frac{3}{4}$ lb. on each bag. Find the cost of the coffee at £4, 7s. 6d. per cwt.

(7) A man owes £4000, but can only pay 12s. $4\frac{1}{2}$d. in the pound. Find the value of his assets.

(8) (*a*) Express ·0463 of £7 + ·256 of 14s. 6d. as a decimal fraction of £15. (*b*) Taking a metre as 39·37 inches, express in yards the difference between 5 miles and 8 kilometres.

(9) Divide £256, 1s. $11\frac{1}{2}$d. between A and B, so that $\frac{3}{5}$ of A's share may equal $\frac{2}{3}$ of B's.

(10) An alloy of silver is mixed with an alloy of gold in the ratio of 7 to 9; the percentage of dross is $13\frac{1}{3}$ in the silver alloy and $17\frac{1}{4}$ in the gold alloy. Find the percentage of dross in the mixture.

XIII.

(1) How many men can be employed for £65458, 6s., if each is to receive £18, 11s. 6d.?

(2) Find the simple interest on £417, 10s. for 7 years and 4 months at $4\frac{1}{4}$ per cent. per annum.

(3) A fortification contains 16800 men and is provisioned for seven months. After 3 months' siege 5600 men break through the enemy's ranks. For how much longer than the seven months will the provisions last?

(4) A dealer buys 750 sheep at £1, 7s. 6d. each, and sells them at £1, 10s. 4d. each. Find the amount of his profit on the transaction. (Express this profit as a percentage of his outlay.)

(5) There are 12 stations on a railway line. Single and return tickets have to be printed from each station to each of the others for first, second, and third class passengers. Find the cost of printing 500 of each kind at 9d. per hundred tickets.

(6) Having given that a metre = 39·37 inches, express in square inches the amount by which a square metre exceeds a square yard.

(7) Find the duty, at 4d. in the pound, payable on 47 tons 13 cwt. 2 qr. 16 lb. of tea.

(8) Find the compound interest on £7250 for $2\frac{1}{2}$ years at 6 per cent.

(9) A dealer marks his goods so as to allow himself 15 per cent. above cost price, but gives his customers a deduction of 5 per cent. off the marked prices for ready money. What percentage of profit does he make on a cash sale?

(10) A publisher paid £85 for printing 10000 copies of a shilling book, £117, 17s. for the paper employed, and $3\frac{1}{4}$d. a copy for binding and other expenses. If he sold 9250 copies at 9d. each, and the rest at 3d. each, find his profit, and express it as a percentage (1) of his outlay, (2) of his receipts.

XIV.

(1) Among how many people can £1000 be divided, so that each shall receive £4, 10s. 6d., and how much money will remain over?

(2) Find to the nearest penny the sum required to pay interest at $2\frac{3}{4}$ per cent. on £768443386.

(3) Three new pennies weigh one ounce avoirdupois. Find in hundredweights, quarters, pounds, and ounces the weight of 12549 new pennies.

(4) A town contains 75600 inhabitants, of whom 51 per cent. are females. Of the females 25 per cent. are married. Find the number of unmarried females in the town without unnecessary calculation.

(5) (*a*) An acre contains 10 square chains, and a chain is 22 yards in length; find the number of square yards in an acre.

(*b*) The chain is divided into 100 equal links. If a field measures 15 chains 25 links in length and 13 chains 46 links in breadth, find its area in square chains.

(*c*) Write down the number of acres in the field, correct to the nearest acre.

(6) Find the cost of repairing a wall 5 furlongs 18 poles 1 yard in length, at the rate of £14, 13s. 4d. per mile.

(7) A dealer buys coal at 19s. per ton, and sells it at 1s. 6d. per cwt. If he sell, on an average, 25 cwt. daily, and his expenses amount to 8s. 6d. a day, find what percentage of his receipts is profit.

(8) A creditor allows his debtors a discount of $2\frac{1}{2}$ per cent., and gives the agent who collects the money $\frac{1}{4}$ per cent. of the sum collected by him. What will the creditor receive out of debts amounting to £6800?

(9) If an article cost 13s. 9d., at what price should it be marked in order that it may be sold at 10 per cent. below the marked price and still bring in a profit of 20 per cent. on the outlay?

(10) A sum of money, lent at $4\frac{1}{2}$ per cent. interest, after deduction of income-tax at 1s. 3d. in £1, yields £71, 14s. $4\frac{1}{2}$d. annually. Find the sum lent.

XV.

(1) Find the simple interest due on a loan of £724, 10s. for $4\frac{1}{2}$ years, the yearly rate of interest being $3\frac{3}{4}$ per cent.

(2) A man left one-third of his property to his wife, one-fourth to his son, one-fifth to his daughter, and £386, 15s. remained for other purposes. Find the amount of the property.

(3) An ocean liner, sailing day and night, covered 2780 nautical miles in 4 days 20 hours. Find its average rate per hour in nautical miles, correct to two decimal places.

(4) If 5 per cent. be gained by selling an article for 18s. $4\frac{1}{2}$d., at what price should it have been sold in order to have gained 10 per cent.?

(5) A metre of wire weighs 43 grams. Find, to the nearest millimetre, the lengths of portions of the same wire which weigh (1) 15 grams; (2) 1 kilogram.

(6) Find, to the nearest penny, the cost of 11 tons 3 cwt. 2 qr. 16 lb., at £12, 16s. 6d. per ton.

(7) Express in acres, roods, and poles, correct to the nearest pole, the area of a rectangular field, whose length is 313 yards and breadth 175 yards.

(8) A grocer buys 3 cwt. of tea for £35, and sells it at the rate of 2s. 3d. per lb. Find his gain *per cent.* on his outlay.

(9) If the 11th October be a Saturday, what day of the week will the following 28th February be?

(10) A cubic centimetre of silver weighs 10·47 grams, a cubic centimetre of gold weighs 19·3 grams. Find the weight of 10 cubic centimetres of an alloy containing one part by weight of silver to eleven parts by weight of gold.

XVI.

(1) Find the value of 8 tons 13 cwt. 24 lb. at £16, 16s. the ton.

(2) If £1095 lying at interest for 117 days becomes £1118, 8s., what is the rate of interest per cent. per annum?

(3) A piece of work can be done in 48 days by 15 men, but after 9 days' work two of the men leave off working. How many additional days will the men who remain require to finish the work?

(4) If a bicyclist ride a mile in 2 minutes 12 seconds, how long will he take to ride a kilometre, assuming that a metre is 3·29 feet?

(5) One bookseller allows his customers 2d. in the shilling discount, and, in addition, 5 per cent. on the remaining price; another simply allows 20 per cent. on the published price of the books. Find which terms are the better for the customer by considering the case when the published price of the books is £6.

(6) A person invests £950 in the $2\frac{3}{4}$ per cents. at 103, brokerage $\frac{1}{8}$ per cent. Find his income for a quarter of a year.

(7) (*a*) Add $1\frac{1}{6}$, $2\frac{3}{16}$, $3\frac{3}{4}$, and divide their sum by the difference between $4\frac{1}{2}$ and $5\frac{1}{3}$. (*b*) Simplify $2{\cdot}879$ of £1 + $9{\cdot}0\dot{6}$ of 1s. + ·24 of 1d.

(8) A goldsmith is required to supply 90 grains of an alloy containing gold and silver in the proportion of 11 to 1. He happens to have an alloy of these metals in the proportion of 15 to 1. How much of this should he take, and how much silver should he add?

(9) A bankrupt's debts amount to £5729, 10s., and his net assets to £893. Find, correct to the nearest penny, how much will be received by a creditor to whom he owes £476, 15s.

(10) The traffic receipts of a railway in 1900 were £1,987,000 and its expenses were £1,144,000. In 1901 it was found that the receipts had fallen $2\frac{1}{2}$ per cent., while the expenses had risen $\frac{1}{2}$ per cent. Find, to two decimal places, by what percentage of itself the profit in 1900 exceeded the profit in 1901.

XVII.

(1) Find the simple interest on £1804, 3s. 4d. for 73 days, at $4\frac{1}{2}$ per cent. per annum.

(2) In a school $\frac{3}{5}$ of the scholars are girls and the rest boys. If $\frac{2}{3}$ of the girls and $\frac{1}{4}$ of the boys be absent, what fraction of the whole number of scholars is present?

(3) If money invested in the $2\frac{3}{4}$ per cents. brings in a return of $2\frac{2}{3}$ per cent., find the price of the stock.

(4) A tea-dealer blends 100 lb. of tea at 1s. 5d. a lb. with 40 lb. at 1s. $8\frac{1}{2}$d. At what price per lb. must he sell the mixture so as to gain 25 per cent.?

(5) Find the value in £, s. d. of 52·5 kilometres of copper wire at ·15 centime per centimetre, taking £1 as equivalent to 25 francs 20 centimes.

(6) A sum of £10,000 is to be divided among several claimants, so that some are to receive £80, 12s. 10d., and the others £80, 12s. 11d. How many persons will receive each of those amounts?

(7) Find, to the nearest penny, the feu-duty of a piece of ground of 2 roods 12 poles 20 yards, at the rate of £20 per acre.

(8) A man bought an estate at 18 per cent. below its real value, and sold it at 10 per cent. above its real value. If his profit was £1358, how much did he pay for the estate?

(9) An insurance company obtains interest on its capital at the rate of £4, 4s. 4d. per cent. per annum. If the amount of the interest be £46,248, 8s., find the capital of the company.

(10) Three partners have capitals of £7400, £6000, and £4600 invested in a business; the first charge on the profits is 5 per cent. interest on capital; after this £200 is paid to each of the partners for management; and the remainder is divided proportionally between the partners. In a certain year the profits were £2310; what did each partner receive?

XVIII.

(1) Express 5 cwt. 2 qr. 21 lb. as a decimal fraction of a ton, correct to 6 places.

(2) What sum will amount to £1513, 6s. 8d. in 3 years at $4\frac{1}{2}$ per cent. simple interest?

(3) A picture, 18 inches broad and $10\frac{1}{2}$ inches deep, is to be photographed on a reduced scale, so that the breadth of

the photograph may be $7\frac{1}{2}$ inches. Find the depth of the photograph.

(4) A grocer sells his best quality of tea at 3s. 6d. per lb., and thereby gains a profit of 4 per cent.; he sells his second quality at 2s. 6d. per lb., gaining a profit of 6 per cent. At what price must he sell a mixture composed of two parts of the second quality to one of the first, so as to gain a profit of 5 per cent.?

(5) A room is 20 feet long and 16 feet 6 inches broad, and it is required to cover all of it, except a margin of 18 inches, with carpet; the roll of carpet is 2 feet 3 inches broad, and costs 4s. 6d. a yard. Find the cost of the carpet required.

(6) How many sums of £1, 17s. $4\frac{1}{2}$d. can be taken from £1000, and what sum will remain?

(7) Find the cost of making a road 12 miles 5 furlongs 121 yards in length, at £86, 13s. 4d. per mile.

(8) If a metre be 39·37 inches, find, to the nearest second, how long a man takes to walk a kilometre at the rate of 4 miles an hour.

(9) Find, correct to the nearest penny, without unnecessary calculation, the value of ·296445 of £1743, 16s. 9d.

(10) The rateable value of a town is £653,784; an assessment has to be made for a charge of £27,846. What will be the necessary rate in the pound, expressed in pence and hundredths of a penny? How much will be collected in excess of the requirement if all the assessment is paid in?

XIX.

(1) Find, by two different methods, the cost of 1440 articles, at £3, 6s. $4\frac{1}{2}$d. each.

(2) What sum will amount to £411, 17s. 9d. in 7 years at $3\frac{1}{2}$ per cent. simple interest?

(3) A man bought 23 baskets of fruit at 4s. 6d. each. Five baskets proved to be bad and were thrown away. Find at what price per basket he must sell the others so as to get back his original outlay and also make a profit of one guinea on the whole transaction.

(4) A railway covers ground to the breadth of 22 yards. How many acres will be covered by a mile of the railway?

(5) If a yard of wire cost a penny, find the value of a kilometre of the same wire, taking a yard as equal to $9\frac{1}{7}$ decimetres.

(6) Find the cost of constructing an electric railway, 6 miles 3 furlongs 25 poles in length, at the average cost of £5, 5s. a yard.

(7) An estate of 612 acres is to be divided among four tenants in proportion to their rentals. If the rents be £189, £207, £297, and £225, how many acres will each tenant receive?

(8) The average profit per annum made in a business during three years was £5744, but during the first two of these years the average profit was £4843, while during the last two it was £6417. Find each separate year's profit.

(9) A tradesman's selling prices are 25 per cent. above cost price, but he allows his customers 5 per cent. on the amount of their bills. What profit per cent. does he make?

(10) A farmer pays rent on 516 acres 3 roods 20 poles at £2, 18s. per acre, but he sublets 52 acres 2 roods 6 poles at £4, 6s. 8d. per acre. Find (1) the net rent he pays; (2) how much this amounts to per acre for the part he retains, correct to the nearest penny.

XX.

(1) Find the simple interest on £128, 19s. 2d. in 5 years, at $4\frac{1}{2}$ per cent.

(2) A takes 10 days to do a piece of work which B can do in 8 days. If A work at it for $3\frac{1}{2}$ days, and is then joined by B, how long will they take to finish it?

(3) A bankrupt's property, after all expenses are paid, amounts to £849, while his debts are £4075. How much will a creditor, to whom he owes £500, receive?

(4) A bronze medal is composed of copper, tin, and zinc. If $\frac{15}{19}$ of the medal be copper and $\frac{1}{8}$ be tin, what fraction of it is zinc?

(5) If a dollar be worth 4s. 2d., and a rupee 1s. $3\frac{1}{2}$d., how many rupees are 186 dollars worth?

(6) What is the cost of 14 tons 2 cwt. 3 qr. 14 lb., at £6, 6s. 8d. per ton?

(7) A grocer buys sugar at 14s. per cwt. and sells it at 2d. per lb. If he lose $2\frac{1}{2}$ lb. out of each cwt., find his gain per cent. on his outlay.

(8) A path, 4 feet wide, is to be made within a rectangular lawn, measuring 45 yards by 40 yards, so as to leave 5 feet between the walk and each side of the lawn. How many square yards will the walk contain?

(9) Multiply 47·23956 by 18·7878, using the contracted method so as to obtain the result, correct to 2 decimal places, without superfluous work.

(10) If the rent of land in France be 125 francs per hectare of 2·47 acres, find the rent per acre in English money, assuming £1 to be worth 25 francs 20 centimes.

XXI.

(1) Amongst how many persons can £249 be divided, if each receive £3, 13s. 2½d., and how much will remain over?

(2) Calculate the simple interest on £3468, 4s. 6d. for 4·25 years at 3·168 per cent.

(3) A debtor who owes £15,040 pays £7238. How much in the pound does he pay?

(4) State as nearly as you can remember how many inches there are in a metre, and how many grains in a gram. Express in kilograms the weight of a kilometre of uniform wire, 25 centimetres of which weigh 41 decagrams.

(5) If 3 per cent. be gained by selling an article for £1, 14s. 4d., at what price must it be sold to gain 5¾ per cent.?

(6) Find, corrected to the nearest penny, the price of 127 tons 16 cwt. 2 qr. of coal, at 24s. 6d. per ton.

(7) What percentage of profit must a trader add to the prime cost of an article in order to show a profit of 15% 'on the return'—that is, on the selling price.

(8) An oblong field is three times as long as it is broad, and its area is 12 acres. Find the cost of fencing it at 2s. 4d. a yard, neglecting fractions of a yard.

(9) What will be the cost of the paper required to print 4000 copies of a book of 304 pages? The paper costs 3¾d. per lb., a sheet contains 16 pages, and a ream of 480 sheets weighs 65 lb.

(10) Find, to two decimal places, how many centimes per kilometre correspond to a penny per mile. (£1 = 25·28 francs. 1 metre = 39·38 inches.)

XXII.

(1) Find the cost of 3 cwt. 2 qr. 11 lb., at £6, 3s. 8d. per cwt.

(2) A man bought a gross of pencils at 6½d. per dozen, a hundred at 1s. 4d. per score, and 44 at 1½d. each. If he then sold them all at 9½d. per dozen, how much did he gain or lose?

(3) A room, 27 feet long, requires 52 square yards of carpet to cover the floor. Find the breadth of the room.

(4) A owes B £505, B owes C £495, C owes D £490, and D owes A £500. Show that they can settle accounts by two payments, and determine the payments.

(5) A bankrupt's debts amount to £9089, 1s. 4d., and his estate realises £5254, 12s. 4d. What can he pay in the pound?

(6) On 1st January 1890 a sum of money was deposited in a bank to receive interest at 3 per cent. per annum. On 4th July the rate of interest was lowered to $2\frac{1}{2}$; but on 14th August it was raised to $3\frac{1}{2}$, and continued at that rate for the rest of the year. Find the average rate of interest for the year.

(7) What is the income of a person whose income-tax, at 7d. in the pound, is £22, 16s. $5\frac{1}{2}$d.?

(8) If a metre be 3·29 feet, how many miles will a man walk in 9 hours 46 minutes 40 seconds at the rate of 6 kilometres an hour?

(9) A sovereign weighs $123\frac{1}{3}$ grains, of which $\frac{11}{12}$ are gold. If gold be $17\frac{1}{2}$ times as valuable as silver, what weight of silver will be equal in value to the gold in 153 sovereigns? Give the answer in pounds and decimals of a pound, avoirdupois.

(10) A manufacturer combines 3 gallons of a mixture which contains 15 per cent. of water, with 2 gallons of one containing 10 per cent. of water, and adds 1 gallon of water. Find the percentage of water in the resulting mixture.

XXIII.

(1) Find the price of 3175 articles at £4, 6s. $10\frac{1}{2}$d. each.

(2) If ·087 of a ton cost £4·71, find the value of 3·4 cwt., expressing the answer in pounds, shillings, and pence, to the nearest penny.

(3) At what rate must simple interest be charged in order that £2375 may increase to £2826, 5s. in four years?

(4) How many yards of paper, 27 inches broad, will be required to cover a wall $3\frac{1}{2}$ yards high and $5\frac{1}{4}$ yards broad?

(5) If 20 square inches be taken as equal to 129 square centimetres, express a square yard as a decimal fraction of a square metre, correct to the nearest digit, in the third significant figure.

(6) Explain how to express a given sum of money at sight as a decimal of £1, taking as examples 13s. 5½d. and 1s. 0¼d., and writing down each as a decimal true to 3 decimal places.

(7) If I can buy 8 apples and 21 oranges for 9d., or 4 apples and 12 oranges for 5d., what are the prices of apples and oranges?

(8) If, by selling sugar at 3d. per lb., a profit of 10 per cent. on the outlay be gained, what will be the gain or loss per cent. by selling it at 26s. per cwt.?

(9) Divide £28, 18s. 10½d. into 3 shares, so that the second share may be $\frac{3}{8}$ of the first, and the third $\frac{4}{5}$ of the second.

(10) A bankrupt estate is expected to yield 18s. in the pound. The assets, however, realise £425 less than the estimate, and the costs of administration amount to 10 per cent. of the dividend distributed, which is 12s. 6d. in the pound. Find the amount of the liabilities.

XXIV.

(1) Find, by practice, the dividend on £7126, 10s., at 9s. 6d. in the pound.

(2) An egg merchant buys eggs at 9d. per dozen, and sells them at 15 for 1s. How much per cent. on his outlay does he gain or lose?

(3) A debtor owes his three creditors £1200, £900, and £300 respectively. If his assets be £850, what should each creditor receive?

(4) If 6 men can reap 24 acres in 3 days, working 12 hours a day, in what time could 9 men, working 10 hours a day, reap a square field, the length of each side being 2 furlongs?

(5) A room is 17 feet long, and 13 feet 6 inches broad. Find (1) how many yards of carpet, 2 ft. 3 in. wide, will be required to cover the floor; (2) the price of the carpet at 5s. 4d. a yard.

(6) The sum of £5000 is deposited with the bank on 1st January 1910, and withdrawn on 27th May the same year. Find the interest gained at 3 per cent. per annum.

(7) An agent buys 17400 tons of coal at 21s. 6d. a ton, and receives a certain percentage of the price as his commission. If the whole amount paid be £18954, 8s., find the rate per cent. at which the commission is allowed.

(8) Three teas are mixed, and the first two amount respectively to 34 and 25 per cent. of the whole. If the weight of the whole be 1 cwt. 2 qr. 13 lb. 4 oz., find the weight of the third tea.

(9) A kilogram weighs 2·20462 lb. avoirdupois. Express the weight of a gram in grains and decimals of a grain.

(10) During a recent year the production of gold was valued at £43,213,000. If one sovereign contains 123·27 grains of gold, and there be 7000 grains in one pound avoirdupois, find the weight in tons of this output, correct to two decimal places.

XXV.

(1) Find the simple interest on £3516, 13s. 4d. for 3 years, at 4½ per cent. per annum.

(2) How many parcels, each weighing 5 lb. 7 oz., can be made up from goods weighing 6 cwt. 2 qr. 27 lb. 13 oz.?

(3) A firm fails and pays 13s. 4d. in the pound on debts amounting to £17,420, 10s. 6d. If the expenses of liquidation amounted to £185, 14s. 4d., find the value of the assets.

(4) The average number of persons admitted to an exhibition during the first five days was 416; the average number during the first six days was 551. How many were admitted on the sixth day?

(5) A piece of material, weighing 50 kilograms, costs £8, 6s. 8d. What is the value of a portion which weighs 6 kilograms 350 grams?

(6) A merchant buys teas at 1s. 6d., 1s. 8d., and 1s. 10d. per lb., and mixes them in the proportion of 5, 4, and 1. At what price per lb. must he sell the mixture so as to make a profit of 25 per cent.?

(7) Find the cost of the paper required for a room 19 feet 9 inches long, 16 feet 3 inches broad, and 11 feet high, if the paper be 2 feet 9 inches broad, and costs 3s. 6d. per piece of 12 yards, assuming that one-sixth of the surface of the walls is not to be papered.

(8) A man spends one-twelfth of his income on life insurance, and on this part he is not charged income-tax. If income-tax, charged on the remainder of his income at the rate of 6d. in the pound, amount to £14, 17s. 11d., find his total income.

(9) The sides of a rectangle are, correct to three decimal places, 8·127 and 13·463 inches. Find the area, correct to two decimal places, and explain why the result cannot be trusted as correct to more than two decimal places.

(10) If £1 be worth 25·18 francs, and 1 lb. avoirdupois be equal to ·4536 kilogram, find to the nearest centime the price of a kilogram of a certain substance in French money, when 1 lb. of the same substance costs 4s. 6d.

XXVI.

(1) I collected £123, 1s. 11d. to be distributed equally to a number of persons. The cost of advertising was £3, 15s. 6d., that of stationery and stamps £2, 13s. 10d., and each person received £5, 1s. 5d. To how many persons was the money distributed?

(2) In what time will £120 amount to £138, 18s. at 4 per cent. simple interest? (Express your answer in years and fraction of a year.)

(3) Five per cent. of an army are killed in battle, and fifteen per cent. of the survivors die of wounds and disease, leaving 18088 men alive. What was the original number of men in the army?

(4) If 124 horses are fed for 7 weeks at a cost of £651, how many horses can be fed for 11 weeks at a cost of £288, 15s.?

(5) The gold reserve of a bank weighs 27 tons 10 cwt. 3 qr. 3 lb. If there be 7000 grains in 1 lb. avoirdupois, and a sovereign weigh 123·374 grains, find the value of the reserve.

(6) A bankrupt's debts amount to £9500. After paying legal expenses, amounting to £138, his assets suffice to pay his creditors a dividend of 4s. 6½d. in the pound. Find the amount of the assets.

(7) The average weight of the crew of an eight-oared boat, including the coxswain, is 10 stones 12 lb., but excluding the coxswain it is 11 stones 2 lb. Find the weight of the coxswain.

(8) Assuming that a 'pint of pure water weighs a pound and a quarter,' and that a gallon contains 277¼ cubic inches, find by how much the weight of a cubic foot of water differs from 1000 ounces. (Express answer in ounces to two places of decimals.)

(9) The interior measurements of a rectangular box are 12 ft. 6 in., 10 ft. 8 in., and 7 ft. 6 in. Find the cost of lining the box

(including the lid) with zinc at 6d. per square foot. Find also the cost, at the same rate, of lining a cubical box which will contain the same bulk of goods.

(10) If American dollars are worth 4s. 7½d. each, and £1 is equivalent to 25·4 francs, find the value of 1000 dollars in francs and centimes.

XXVII.

(1) Find the interest on £500 for 84 days at $6\frac{1}{2}$ per cent. per annum.

(2) A newspaper consists of 10 pages, each of which contains 7 columns with 185 lines in each column. If there be, on an average, 9 words in each line and 4½ letters in each word, find, as nearly as possible, the number of letters in the whole newspaper.

(3) A grocer bought a box of tea containing 3 cwt. 2 qr. 22 lb. for £36, and sold the tea at 2s. the pound. Find (1) the amount of his profit; (2) the gain per cent. on his outlay.

(4) Find the total rental of a town if a tax of 9d. in the pound on rents brings in £2163, 14s. 3d.

(5) How many meshes are there in a square foot of wire gauze, each mesh being ·3 of an inch long and ·1 of an inch wide?

(6) (*a*) Find the value of ·21875 of £2, 10s. (*b*) Reduce 15 cwt. 3 qr. 20 lb. to the decimal of 10 tons, correct to 3 decimal places.

(7) New sovereigns weigh 123·27 grains, and are allowed to remain in circulation till their weight is reduced to 122·5 grains. A thief obtained new sovereigns and reduced them to the latter weight, thereby retaining gold worth £231. How many sovereigns did he reduce in weight?

(8) Multiply 47·23956 by 18·7878, using the contracted method so as to obtain the result, correct to 2 decimal places, without superfluous work.

(9) On 1st January A started in business with a capital of £8000. On 1st May B joined him as a partner, contributing £7000 on the understanding that A received 20 per cent. of the profits in lieu of salary as manager, in addition to his share as partner. If the profits at the end of the year were £1330, how much should each receive?

(10) A merchant buys, in Paris, velvet at 5 francs per metre. At

what price per yard must he sell it in London so as to make 10 per cent. profit on his outlay?

Exchange in Paris, £1 = 25·20 francs.

The metre to be taken as 39·37 inches.

XXVIII.

(1) £135, accumulating at simple interest, becomes £189 in 12 years. Find the rate per cent. per annum.

(2) What sum will remain out of 25 guineas, after paying 8s. 6d. for bread, £2, 15s. 3½d. for meat, 19s. 6d. for fish, 8s. 10½d. for vegetables, £1, 16s. 8d. for groceries, £1, 4s. 6½d. for milk, 14s. 8d. for washing, £3, 14s. 6d. for dress, £4, 13s. 6d. for wages, and £5, 8s. 4d. for rent?

(3) An army of 145,600 men loses 12½ per cent. of its number through disease, and 15 per cent. of the remainder in battle. How many men are left?

(4) Divide £100 among A, B, and C, so that A may have £20 more than B, and £30 more than C.

(5) A person borrowed a sum of money at 3½ per cent., and invested it in a stock at 115 paying 4½ per cent. dividend. If he gained thereby £190 in 11½ years, how much money did he borrow?

(6) A bankrupt has assets £600, and liabilities £939; of which latter amount £35 are preferential claims for rent, taxes, &c., and must be paid in full. Find how much in the pound the creditors ought to receive.

(7) Find the value of a cubic foot of gold which weighs 1210 lb. avoirdupois, if a sovereign be worth 113 grains of pure gold, and 1 lb. avoirdupois contain 7000 grains.

(8) The cost of producing 5000 copies of a book is estimated to be £475. If 60 presentation copies are given away, and on the rest the author is to receive a royalty of 2d. per copy, at what price per dozen (giving 13 copies in the dozen) must the publisher sell the remaining copies so as to make a profit of 15 per cent.?

(9) If the pressure of air in a diving-bell be 68 lb. 1 oz. on a square inch, express that pressure in grams on a square centimetre, taking an inch as $2\frac{7}{13}$ centimetres and 10 lb. as $4\frac{7}{13}$ kilograms.

(10) Prove the following rule for finding any percentage of a given whole number of pounds sterling: *Multiply the number of*

pounds by twice the rate per cent., set off one decimal place, and the result represents in shillings the percentage required. Exemplify this rule by making use of it to find (1) $3\frac{1}{2}$ per cent. of £25; (2) $4\frac{1}{2}$ per cent. of £1247.

XXIX.

(1) Express (1) £2, 1s. 3d. as a vulgar fraction of £2, 11s. 4d.
(2) 5 cwt. 2 qr. 7 lb. as a decimal fraction of a ton.

(2) A man borrowed £280 and repaid it 9 months later. Find the simple interest due at $4\frac{3}{4}$ per cent. per annum.

(3) A man mixes 1 cwt. of tea at 1s. 7d. per lb. with 2 cwt. 14 lb. at 2s. per lb. What is the value of 5 lb. of the mixture?

(4) A man holding £4850 of 4 per cent. stock sells it at 133, and invests the proceeds in $2\frac{3}{4}$ per cent. stock at $104\frac{1}{2}$. Find the change in his income, assuming that there is no charge for brokerage.

(5) A steam-engine, working steadily day and night, consumes a hundredweight of coal in $7\frac{1}{2}$ hours. How many tons will be used in 100 days?

(6) A bankrupt's debts amount to £10,659, 5s., and his effects, when realised, amount to £888, 5s. 5d. Find (1) what rate of dividend per pound he can pay; (2) how much a creditor for £150 will lose.

(7) A boy spends ·125 of his money in one shop and ·875 of the rest in another. If he has $10\frac{1}{2}$d. left, how much had he at first?

(8) If a 4 lb. loaf be worth 6d., find, to the nearest centime, the value of a loaf of 2 kilograms, assuming that a kilogram $=2{\cdot}205$ lb., and that £1 = 25 francs 30 centimes.

(9) Determine, *without unnecessary calculation*, the number of millions (correct to the nearest million) in the product of 719386 and 28763.

(10) A man's income is £300 a year. He pays no income-tax on the first £160, and 9d. in the pound on the rest. His house-rent is £43, 3s. 3d., and he spends otherwise at the rate of 10s. $3\frac{1}{2}$d. a day. How much will he save in the year 1912?

XXX.

(1) The rate of bank interest is raised from $3\frac{1}{4}$ to $3\frac{1}{2}$ per cent., and a depositor finds that his interest is increased by 1s. $5\frac{1}{2}$d. per month. Find the amount of his deposit.

(2) A man owes his creditors £2656, but can only pay them 8s. 7½d. in the pound. How much money has he?

(3) A man's annual income is £483, 17s., and his average daily expenditure during the year 1910 is £1, 2s. 4d. How much will he save in that year?

(4) There are 156 boys in the junior and 162 in the senior department of a school. If the boys constitute 65 per cent. of the scholars in the junior department and 45 per cent. of those in the senior department, what percentage are they of the whole school?

(5) If a metre be 39·37 inches, express an inch in millimetres, correct to four significant figures.

(6) If money invested in the 3½ per cents. brings in 3 per cent., find (*a*) the price of the stock, and (*b*) the amount of stock which can be bought for 1000 guineas.

(7) Make out a bill for:

47 lb. 4 oz. of tea, at 1s. 10d. per lb.;
38 lb. 8 oz. of sugar, at 2½d. per lb.;
18 yd. 18 in. of linen, at 8½d. per yd.;
282 boxes of matches, at 2½d. per dozen;

deducting 5 per cent. for ready-money payment.

(8) Six months' working of a tramway showed a return of 314464 passengers carried, at an average fare of 1⅜d. If the profits were £431, 4s. 7d., find, to two decimal places, the percentage of receipts absorbed by expenses.

(9) A mortgage bond yields interest at the rate of 3½ per cent., which, after deduction of income-tax at 1s. 2d. in the pound, is divided among A, B, and C, in the proportion of 3, 4, and 5. If A receive £6, 11s. 10d. as his share for a year, what is the sum which has been lent on mortgage?

(10) Obtain the following values, correct to 3 decimal places, without unnecessary calculation:

(1) 628·875037 × 5·963855.
(2) 1572·21 ÷ 4971·14.

XXXI.

(1) In a division sum the divisor is 8274, the quotient is 94,907, and the remainder 65. What is the dividend?

(2) Among how many people must a sum of £420, 11s. 10½d. be divided so that each may receive £3, 5s. 2½d.?

(3) Divide 4·3046721 by ·0729, and find the value of 3·16875 of £1.

(4) A person, having £5, pays away ·05 of his money, and then ·15

of the remainder. Express the money he now has as a decimal of £10.

(5) If, during the year 1912, a man were to receive 25s. per week for 50 weeks' work, and were to spend, during the whole year, average sums of 1s. 1½d. per day for food, 8s. 9d. per week for lodgings, and 29s. per calendar month for other expenses, how much would he save? (Calculate the charge for the lodgings for the whole year, and not merely for the number of exact weeks.)

(6) A grocer bought rice at £1, 4s. 6d. per cwt., and sold it at 3½d. per lb. What was his gain per cent.?

(7) Find the cost of carpeting a room 17 ft. by 13½ ft. with carpet 2 ft. 3 in. wide and costing 5s. 4d. a yard.

(8) A tradesman marks his goods 25 per cent. above cost price, but allows a customer 4 per cent. discount. What is the tradesman's net profit per cent.?

(9) A man borrows £1000, interest to be payable at the rate of 4 per cent. per annum. At the end of each year he pays £100, the balance after payment of interest being applied to the reduction of the debt. Find, to the nearest penny, the amount of his debt immediately after he has made his fourth payment.

(10) A man invests £840 in the $2\frac{3}{4}$ per cents. at 99, and £1470 in the 4 per cents. at 126. Find the average rate of interest per cent. which he obtains.

XXXII.

(1) When the number 2566524 is divided by a certain divisor, the quotient is 875, and the remainder 149; find the divisor.

(2) State and prove the rule for the multiplication of two decimal numbers. Simplify—

(a) $4{\cdot}93 \times 3{\cdot}571 \times 1000$.

(b) $(\cdot0005)^2 \div (\cdot002)^3$.

(3) (a) Express ·0390625 of a day (24 hours) in minutes and seconds.

(b) If 8 metres are equal to 315 inches, express 1 metre as a decimal fraction of a yard.

(4) Find the value of a property when a person who possesses $\frac{3}{7}$ of it sells $\frac{4}{5}$ of his share for £620, 8s.

(5) If 20 square inches be taken as equal to 129 square centimetres, express a square yard as a decimal fraction of a square metre, correct to the nearest digit, in the third significant figure.

(6) How many acres are contained in a right-angled triangular field, if the sides containing the right angle are 37 chains 50 links and 16 chains respectively?

(7) Find, correct to the nearest penny, the compound interest on £7486, 10s. for 5 years at 3 per cent.

(8) How much tea at 2s. and 3s. per lb. respectively must a grocer mix so as to have 48 lb. worth 2s. 4d. per lb.?

(9) Express the following in British money: (*a*) £6, 5 fl.; (*b*) £3, 9 fl. 5 c.; (*c*) £4·025; (*d*) £8, 5 fl. 3 c. 5 m.

(10) On a certain day the prices of $2\frac{1}{2}$ and $2\frac{3}{4}$ per cent. stocks are respectively $94\frac{1}{4}$ and $96\frac{1}{8}$, including brokerage. Find which stock will give the larger return, and calculate, to the nearest penny, the difference in annual income, if the sum invested were £10,000.

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